LG公式单

定&不定积分 - 句话: 「eg. $xy'=y(\ln x-\ln y)$

$y'=f(u)$; $u=\frac{y}{x}$

或换 $\xi=\ln x$, $\eta=\ln y$.」

看谁不顺眼把谁换掉. 技: 〈 夏变 / 双曲

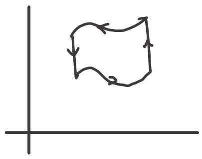
高数下: | 曲面积分: $dS=\sqrt{1+f_x^2+f_y^2}\,d\sigma$

$=\sqrt{EG-F^2}\,du\,dv$ 「$E=x_u^2+y_u^2+z_u^2$, $G=x_v^2+y_v^2+z_v^2$, $F=x_ux_v+y_uy_v+z_uz_v$」

Green. Stokes. Gauss 公式 $\Rightarrow$ 外微分.

Stokes 条件: P.Q 有连续一阶偏导.

证法: ① | $ds\geq 0$. eg $ds=|dx|$. 记得取模.



化为二重积分

② 单连通拼接.

③ 多连通.

Gauss 条件. $\oint \frac{\partial u}{\partial n}\cdot ds=\iint \Delta u\, d\sigma$.

u有连续二阶偏导.

证法: (i) 方向向量. + 方向导数定义. (ii) 同Stokes

技巧: $\vec{n}_0\cdot\vec{\tau}=0$

$\vec{n}_0\times\vec{\tau}=\vec{k}$ 模1.

〈 $\oint=0 \Rightarrow$ 与路径无关

小心无定义的点. (去掉单算).

换积分变量记得加 Jacobi

$P.Q\in C^1[D]$

$\frac{\partial Q}{\partial x}=\frac{\partial P}{\partial y} \Leftrightarrow Pdx+Qdy$ 为全微分.

① 充分: 构造 $u=\int Pdx+Qdy.\ (\oint=0)$

验证 $\partial_x u$, $\partial_y u$

② 必要: 求导 + 二阶偏导可交换

微分方程: $y''=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{1}{2}\frac{d}{dy}(y'^2)$

1. 分离变量.

2. (一阶)齐次方程. 换元 $p=\frac{y}{x}$. $y=xp$

$f(p',p,x)=0$.

3. 常数变易.

「二阶常数变易:

$y''+P_1(x)y'+P_2(x)y=q(x)$

两齐次通解: $y_1(x)$. $y_2(x)$.

令 $y=C_1(x)y_1(x)+C_2(x)y_2(x)$.

$y'=C_1'(x)y_1(x)+C_2'(x)y_2(x)$

$+C_1(x)y_1'(x)+C_2(x)y_2'(x)$

为避免常数出二阶项. 人为令:

$C_1'(x)y_1(x)+C_2'(x)y_2(x)=0$.

$\Rightarrow y''=C_1'(x)y_1'(x)+C_2'(x)y_2'(x)$

$C_1(x)y_1''(x)+C_2(x)y_2''(x)$

$$\Rightarrow \begin{bmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{bmatrix}\begin{bmatrix} C_1'(x) \\ C_2'(x) \end{bmatrix}=\begin{bmatrix} 0 \\ q(x) \end{bmatrix}$$

4. 特征根. 重根加 x^m Exp.

5. 熟记积分因子:

常用: x^my^n, $(\alpha x+\beta y)$, $e^{\mu x}$.

6. $Pdx+Qdy=0$ 〈 天然全微分 $\partial_yP=\partial_xQ$

$\partial_xP=0$ / $\partial_yP=0$ 有技巧

appendix: Wronski

「* Integrate $\frac{1}{x}\rightarrow \ln|x|$.

别忘加绝对值!」

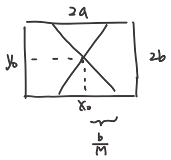
注意顺序.

△ 李普希茨条件: 在D中 $\exists L>0$ $\forall x$. s.t. $\forall y_1 y_2$

$$|f(x,y_1)-f(x,y_2)| \le L|y_2-y_1|$$

用途: 皮卡序列收敛

皮卡:



$M=\max\{|f(x,y)|\ |(x,y)\in R\}$.

在 $\{y_0 \pm b,\ x_0 \pm h\}$. $h=\min\{\frac{b}{M},a\}$

$y'=f(x,y)$ 有唯一解

△ 微分方程求解.

$$\sum_{i=1}^{n} a_{m_i} x^{m_i} y^{(m_i)} = 0 \Rightarrow \text{换元 } x=e^t$$

△ 无穷级数:

* 柯西收敛原理 (序列)

$\forall \varepsilon>0$ $\exists N>0$. $\forall\ m,n>N$ 有 $|a_n-a_m|<\varepsilon$.

(函数) $f(x)$

$\forall \varepsilon>0$ $\exists \delta>0$, $\forall x_1, x_2$ $0<|x_i-a|<\delta$

有 $|f(x_1)-f(x_2)|<\varepsilon$.

* 数项级数: $\sum a_n$

收敛必要条件: $a_n \to 0$ $(n\to\infty)$

充要条件: $\forall \varepsilon>0$. $\exists N$ $\forall n \ge N$, $p\ge 1$.

$$\left|\sum_{k=n+1}^{n+p} a_k\right| < \varepsilon.$$

性质: 可加, 可乘(常系数). 不等式取极限. $\forall$加括号(结合律).

正项级数

① 收敛 $\Leftrightarrow$ 有上界

可乘常数与比较

方法: 比较判别法 ($u_n \le v_n \to u_n \le c\,v_n$)

→ 比较法拓展 $\lim\limits_{n\to\infty} \frac{u_n}{v_n} = h$ $\begin{cases} h\in[0,+\infty) \\ h\in(0,+\infty] \end{cases}$

达朗贝尔: 正项级数 $\sum u_n$

$\lim\limits_{n\to\infty} \frac{u_{n+1}}{u_n} = l$

$l<1$ 敛

$l>1$ 散

$l=1$ 不定

柯西: $\lim\limits_{n\to\infty} \sqrt[n]{u_n} = l$

$l<1$ 敛

$l>1$ 散

$l=1$ 不定

拉阿伯 $\lim\limits_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)=R$

$R>1$ 敛 ← (R可以$=\infty$.)

$R<1$ 散

$R=1$ 不定

若 $\exists$ 单调下降的非负 $f(x)$. $u_n=f(n)$

则 $\sum u_n$ 与 $\int_1^{\infty} f(x)\,dx$ 同敛散

函数级数.

$\sum u_n(x) \rightrightarrows S(x)$ 则 $u_n(x) \rightrightarrows 0$ $(n\to\infty)$

· 柯西. 将原来的 "$\to$" 变为 "$\rightrightarrows$"

· 强级数 $|u_n(x)| \le a_n$. 由 $\sum a_n$ 敛 $\Rightarrow$ $\sum u_n(x)$ 一致敛

性质: 一致收敛 + u_n 连续 $\Rightarrow$ $S(x)$ 连续.

一致收敛 + u_n一致连续 $\Rightarrow$ $S(x)$ 一致连续.

△ 补充: 若要证明级数在开区间内连续, 即只能证明函数在开区间收敛. 同时有任一闭区间一致收敛, 则可以利用局部一致收敛证明原级数在开区间内连续

☆ 狄: $\forall x$ $a_n(x)$ 对 n 单调 $\to 0$.

$\sum_{n=1}^{\infty} b_n(x)$ 的部分和序列一致有界

$\Rightarrow \sum_{n=1}^{\infty} a_n(x) b_n(x)$ 一致收敛.

☆ 阿: $\forall x$ $\{a_n(x)\}$ 单调. 一致有界.

$\sum_{n=1}^{\infty} b_n(x)$ 一致收敛.

$\Rightarrow \sum_{n=1}^{\infty} a_n(x) b_n(x)$ 一致收敛.

逐项求积分:

一致收敛 + $u_n(x)$ 连续

$\Rightarrow$ $S(x)=\sum u_n(x)$ 可积, 且可逐项积.

逐项求导:

$\sum^{\infty} u_n(x)$ 点点收敛. $u_n'(x)$ 在 $[a,b]$ 连续.

$\sum^{\infty} u_n'(x)$ 一致收敛 $\Rightarrow$ $S(x)$ 可导、可逐项导. $S'(x)$ 连续.

幂级数

* $\forall$ 幂级数. 收敛半径存在唯一. $\begin{cases} (-R,R) \text{ 绝对收敛} \\ \text{闭区间一致收敛 且一致连续} \\ (u_n=a_nx^n \text{ 自动连续}) \end{cases}$

(若端点收敛, 则含端点闭区间).

* $\sum_{n=0}^{\infty} a_n x^n$ $\lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = l$ $\begin{cases} \text{if} & 0<l<+\infty & R=1/l \\ \text{elif} & l=0 & R=+\infty \\ \text{elif} & l=+\infty & R=0 \end{cases}$

p.s. 当 $\left|\frac{a_{n+1}}{a_n}\right|$ 极限不存在. 使用比较判别法 $\left|\frac{u_{n+1}}{u_n}\right| = \square\, |x|^m$

如 $\sum \frac{x^{n^2}}{2^n}$

幂函数可积, 且逐项可积. 可导且逐项可导.

(开区间内).

无穷积分:

柯西: $\forall \varepsilon$. $\exists A_0>a$ $A, A'>A_0$ 则:

$\left|\int_A^{A'} f(x)\,dx\right| < \varepsilon \Rightarrow \int_a^{+\infty} f(x)\,dx$ 收敛.

p.s. 若 $\int_a^{+\infty} |f(x)|\,dx$ 收敛 $\Rightarrow \int_a^{+\infty} f(x)\,dx$ (绝对)收敛.

→ 条件收敛: 收敛. 但不绝对收敛.

比较判别法. $0\le f(x) \le g(x)$

"高级版本"

极限形式: $\lim\limits_{x\to+\infty} \frac{f(x)}{g(x)} = k$ [$g(x)\neq 0$ 条件下使用]

① $k\in[0,+\infty)$. ② $k\in(0,+\infty]$. …

狄氏, 阿贝尔 判别.

含参广义积分.

* 一致收敛: $\forall\varepsilon\ \exists A>a$, 对 $\forall y$. $\left|\int_A^{+\infty} f(x,y)\,dx\right| < \varepsilon$.

则称含参无穷积分 $\int_a^{+\infty} f(x,y)\,dx$ 在 Y 上一致收敛

* 柯西: $\forall\varepsilon$, $\exists A_0>a$. 对 $A, A' > A_0$ 有

$\left|\int_A^{A'} f(x,y)\,dx\right| < \varepsilon$ 　则一致收敛

* 强收敛: 若 当 $x>a$ 时. 对 $\forall y$.

$|f(x,y)| \le \varphi(x)$

且 $\int_a^{+\infty}\varphi(x)\,dx$ 收敛. 　则 $\int_a f(x,y)\,dx$ 一致收敛.

* 狄: $g(x,y)$ 在 x 充分大后 单调 $\to 0$

$\forall A>a$ 　$\int_a^A f(x,y)\,dx$ 存在且一致有界.

则 $\int_0^{+\infty} f(x,y)\,g(x,y)\,dx$ 一致收敛.

* 阿: $g(x,y)$ 在 x 充分大后 单调有界.

$\forall A>a$ 　$\int_a^A f(x,y)\,dx$ 存在 且一致收敛.

则 $\int_0^{+\infty} f(x,y)\,g(x,y)\,dx$ 一致收敛.

连续性: 含参广义积分同 函数项级数类似.

一致收敛 + 连续 $\Rightarrow$ 区间内连续.

任一闭区间一致收敛 + 连续 $\Rightarrow$ 开区间连续.

积分: $f(x,y)$ 在 $[a,+\infty)\times[c,d]$ 上连续

且 $g(y) = \int_a^{+\infty} f(x,y)\,dx$ 一致收敛

$\Rightarrow$ $g(y)$ 连续, 可积, $\int_c^d dy\int_a^{+\infty}dx\, f(x,y)$ 可换序.

微分: $f(x,y)$, $f_y(x,y)$ 在 $[a,+\infty)\times[c,d]$ 上连续

$g(y) = \int_a^{+\infty} f(x,y)$ 在 $[c,d]$ 点点收敛.

$\int_a^{+\infty} \partial_y f(x,y)\,dx$ 一致收敛.

$\Rightarrow$ $g(y)$ 在 $[c,d]$ 上可导.

$g'(y) = \int_a^{+\infty} f_y(x,y)\,dx$

二元积分: $f(x,y)$ 连续.

$\int_a^{+\infty} f(x,y)\,dx$ 及 $\int_c^{+\infty} f(x,y)\,dy$ 一致收敛

且 $\int_a^{+\infty}dx\int_c^{+\infty}dy\,|f(x,y)|$ 及 $\int_c^{+\infty}dy\int_a^{+\infty}dy\,|f(x,y)|$ 之一存在

则: $\int_a^{+\infty}dx\int_c^{+\infty}dy\, f(x,y) = \int_c^{+\infty}dy\int_a^{+\infty}dx\, f(x,y)$ 存在且相等.

Γ函数. B函数.

$$\Gamma(z) = \int_0^{+\infty} t^{z-1}e^{-t}\,dt$$

$$\Gamma(z+1) = z\Gamma(z)$$

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$$

$$\Gamma(n+1) = n! \quad (n\in\mathbb{N})$$

$$\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin\pi z}$$

$$\Gamma(2z) = 2^{2z-1}\pi^{-1/2}\,\Gamma(z)\Gamma(z+\tfrac{1}{2})$$

$$B(p+1,q) = \frac{p}{p+q}B(p,q)$$

$$B(p,q+1) = \frac{q}{p+q}B(p,q)$$

$$\begin{aligned}B(p,q) &= \int_0^1 x^{p-1}(1-x)^{q-1}\,dx\\ &= 2\int_0^{\pi/2}\sin^{2p-1}x\cos^{2q-1}x\,dx\\ &= \int_0^{+\infty}\frac{t^{p-1}}{(1+t)^{p+q}}\,dt\\ &= \int_0^{+\infty}\frac{e^{-pt}}{1+e^{-t}}t^{q-1}\,dt\\ &= \frac{1}{p}\cdot\cfrac{1}{1+\frac{q-1}{p+1}\cdot\cfrac{1}{1+\frac{q-1}{p+2}\cfrac{1}{1+\frac{q-1}{p+3}\cdot\frac{1}{1+\cdots}}}}\end{aligned}$$

$$B(p,q) = B(q,p) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

$$B(p,q) = B(p+1,q) + B(p,q+1)$$

Fourier 级数.

「充分条件: 含有限个第一类间断点.」

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx) = g(x)$$

$$g(x) = \frac{1}{2}[f(x-0) + f(x+0)].$$

$$\frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2) = \frac{1}{\pi}\int_{-\pi}^{\pi} f^2(x)\,dx$$

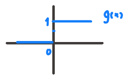
① $f(x) = x \sim \sum_{n=1}^{\infty}(-)^{n+1}\cdot\frac{2}{n}\sin nx$

取 $x=1$ $\Rightarrow$ $\sum_{n=1}^{\infty}(-)^n\frac{\sin n}{n} = -\frac{1}{2}$ # 　$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$

② $f(x) = |x| \sim \frac{\pi}{2} + \sum_{k=1}^{\infty} -\frac{4}{(2k-1)^2\pi}\cos(2k-1)x$

$\Rightarrow$ $\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2} = \frac{\pi^2}{8}$ 　$\sum_{k=1}^{\infty}\frac{1}{(2k-1)^4} = \frac{\pi^4}{96}.$

③ $f(x) = \eta(x) \sim \frac{1}{2} + \sum\frac{2}{\pi}\frac{\sin(2k-1)x}{2k-1}$



$\Rightarrow$ $\sum_{k=1}^{\infty}\frac{\sin(2k-1)}{2k-1} = \frac{\pi}{4}$ # $\overset{由\#}{\Rightarrow}$ $\sum_{k=1}^{\infty}\frac{\sin n}{n} = \frac{\pi-1}{2}$

$\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2} = \frac{\pi^2}{8}$

④ $f(x) = \mathrm{sgn}(x) \sim \sum_{k=1}^{\infty}\frac{4}{\pi}\frac{\sin(2k-1)x}{2k-1}$

$\Rightarrow$ $\sum_{k=1}^{\infty}\frac{\sin(2k-1)}{2k-1} = \frac{\pi}{4}$. 　$\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2} = \frac{\pi^2}{8}$

⑤ $f(x) = x^2 \sim \frac{\pi^2}{3} + \sum(-)^n\frac{4}{n^2}\cos nx$

令 $x=0$ $\Rightarrow$ $\sum_{n=1}^{\infty}(-)^n\frac{1}{n^2} = -\frac{\pi^2}{12}$ 　$\sum(-)^n\frac{\cos n}{n^2} = \frac{1}{4}-\frac{\pi^2}{12}$

$\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{8}{45}\pi^4.$

⑥ $f(x) = x^2\mathrm{sgn}(x) \sim \sum\left[\frac{\pi}{n}(-)^{n+1} + \frac{4}{n^3\pi}((-)^n-1)\right]\sin nx$

$\Rightarrow$ $\sum(-)^n\frac{\sin n}{n} + \sum\frac{4\sin(2k-1)}{\pi(2k-1)^3} = 1$ $\Rightarrow$ $\sum\frac{\sin(2k-1)}{(2k-1)^3} = \frac{\pi}{8}(1-\pi)$

终结数项级数:

$\sum_{n=1}^{\infty}(-)^n\frac{\sin n}{n} = -\frac{1}{2}$, $\sum_{n=1}^{\infty}(-)^n\frac{\sin n}{n} = -\frac{1}{2}$. $\sum_{n=1}^{\infty}(-)^{n-1}\frac{1}{n^2} = \frac{\pi^2}{12}$

$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$. $\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2} = \frac{\pi^2}{8}$. $\sum_{n=1}^{\infty}\frac{1}{(2n)^2} = \frac{\pi^2}{24}$

$\sum_{k=1}^{\infty}\frac{1}{(2k-1)^4} = \frac{\pi^4}{96}$, $\sum_{k=1}^{\infty}\frac{\sin(2k-1)}{2k-1} = \frac{\pi}{4}$ 　$\sum_{n=1}^{\infty}\frac{\sin n}{n} = \frac{\pi-1}{2}$

$\sum_{k=1}^{\infty}\frac{\sin(2k)}{2k} = \frac{\pi}{4}-\frac{1}{2}$. $\sum_{n=1}^{\infty}(-)\frac{\cos n}{n^2} = \frac{1}{4}-\frac{\pi^2}{12}$. $\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{8}{45}\pi^4.$

$\sum_{k=1}^{\infty}\frac{\sin(2k-1)}{(2k-1)^3} = \frac{\pi}{8}(1-\pi)$, $\sum_{k=1}^{\infty}\frac{\cos n\varphi}{n^2} = \frac{2\pi^2-6\pi\varphi+3\varphi^2}{12}$

级数 *

计算积分 $\int_{-\infty}^{+\infty}\frac{\sin x}{x}dx$

$f(x)=\frac{\sin x}{x}$ 为偶函数. ∴ $\int_{-\infty}^{+\infty}\frac{\sin x}{x}dx = 2\int_0^{+\infty}\frac{\sin x}{x}dx$

法一: 定义 $I(\alpha)=\int_0^{+\infty}\frac{\sin x}{x}e^{-\alpha x}dx$

则 $I'(\alpha)=\frac{d}{d\alpha}\int_0^{+\infty}\frac{\sin x}{x}e^{-\alpha x}dx$

$=\int_0^{+\infty}\frac{\partial}{\partial\alpha}\left(\frac{\sin x}{x}e^{-\alpha x}\right)dx$

$=-\int_0^{+\infty}\sin x\, e^{-\alpha x}dx$

$=\frac{1}{\alpha}\left(\sin x\, e^{-\alpha x}\Big|_0^{+\infty} - \int_0^{+\infty} e^{-\alpha x}\, d\sin x\right)$

$=-\frac{1}{\alpha}\int_0^{+\infty}e^{-\alpha x}\cos x\, dx$

$=\frac{1}{\alpha^2}\int_0^{+\infty}\cos x\, de^{-\alpha x}$

$=\frac{1}{\alpha^2}\cos x\, e^{-\alpha x}\Big|_0^{+\infty} - \frac{1}{\alpha^2}\int_0^{+\infty}e^{-\alpha x}\, d\cos x$

$=-\frac{1}{\alpha^2}+\frac{1}{\alpha^2}\int_0^{+\infty}e^{-\alpha x}\sin x\, dx$

故 $\left(1+\frac{1}{\alpha^2}\right)I'(\alpha)=-\frac{1}{\alpha^2}$

$I'(\alpha)=-\frac{1}{\alpha^2+1}$

∴ $I(\alpha)=\int I'(\alpha)\,d\alpha = -\arctan\alpha + C$

观察 $I(\alpha)$. $\alpha\to+\infty$ $I(\alpha)\to 0$ ∴ $C=\frac{\pi}{2}$

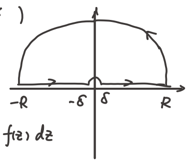
∴ $I(0)=\frac{\pi}{2}$

∴ $\int_{-\infty}^{+\infty}\frac{\sin x}{x}dx = 2I(0)=\pi$

法二: $\int_{-\infty}^{+\infty}\frac{\sin x}{x}dx$

$=\frac{1}{2i}\left(\int_{-\infty}^{+\infty}\frac{e^{ix}}{x}dx - \int_{-\infty}^{+\infty}\frac{e^{-ix}}{x}dx\right)$

第一项: $f(z)=\frac{e^{iz}}{z}$



则 $\oint_C f(z)dz = \int_{C_\delta}f(z)dz+\int_\delta^R f(z)dz+\int_{C_R}f(z)dz + \int_{-R}^{-\delta}f(z)dz$

$\int_{-\infty}^{+\infty}f(z)dz = \lim_{\substack{\delta\to0\\ R\to\infty}}\int_\delta^R f(z)dz+\int_{-R}^{-\delta}f(z)dz$

C 内无奇点. $\oint_C f(z)dz=0$.

$\lim_{z\to0} z f(z)=1$ $\lim_{z\to\infty}\frac{1}{z}=0$

由 小圆弧. Jordan 引理.

$\lim_{\delta\to0}\int_{C_\delta}f(z)dz=-i\pi$ $\lim_{R\to\infty}\int_{C_R}f(z)dz=0$

∴ $\int_{-\infty}^{+\infty}f(z)dz = i\pi$

第二项: $g(z)=\frac{e^{-iz}}{z}$

$\oint_C g(z)dz = \int_{C_\delta}g(z)dz+\int_\delta^R g(z)dz+\int_{C_R}g(z)dz+\int_{-R}^{-\delta}g(z)dz$

$\lim_{z\to0} z g(z)=1$ $\lim_{z\to\infty}\frac{1}{z}=0$

由 小圆弧. 补充引理. z=0 为奇点.

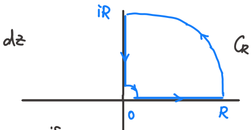
$\lim_{\delta\to0}\int_{C_\delta}g(z)=-i\pi$ $\int_{C_R}g(z)dz+\int_{C'_R}g(z)dz=\oint_{C'}g(z)dz$ (下半平面)

$\lim_{R\to\infty}\int_{C_R}g(z)dz = \oint g(z)dz - \int_{C'_R}g(z)dz = 2\pi i$

∴ $\int_{-\infty}^{+\infty}g(z)dz=-\pi i$

∴ $\int_{-\infty}^{+\infty}\frac{\sin x}{x}dx = \frac{1}{2i}(\pi i-(-\pi i))=\pi$

法三: $\int_{-\infty}^{+\infty}\frac{\sin x}{x}dx = -2\,\mathrm{Im}\int_0^{+\infty}\frac{e^{iz}}{z}dz$



$f(z)=\frac{e^{iz}}{z}$

$\oint_C f(z)dz = \int_\delta^R f(z)dz + \int_{C_R}f(z)dz+\underbrace{\int_{iR}^{i\delta}f(z)dz}_{\mathrm{Im}=0} + \int_{C_\delta}f(z)dz$

$\lim_{R\to\infty}\int_{C_R}f(z)dz=0$ $\lim_{\delta\to0}\int_{C_\delta}f(z)dz=-i\cdot\frac{\pi}{2}$

∴ $\mathrm{Im}\int_0^{+\infty}\frac{e^{iz}}{z}dz=-\frac{\pi}{2}$

∴ $\int_{-\infty}^{+\infty}\frac{\sin x}{x}dx=\pi$

法四: 我们知道 $\frac{f(t)}{t} \doteq \int_p^{+\infty}F(q)\,dq$

即 $\int_0^{+\infty}e^{-pt}\frac{f(t)}{t}dt = \int_p^{+\infty}F(q)\,dq$

取 $p=0$ ⇒ $\int_0^{+\infty}\frac{f(t)}{t}dt = \int_0^{+\infty}F(q)\,dq$

$\sin x \doteq \frac{1}{1+p^2}$

∴ $\int_0^{+\infty}\frac{\sin x}{x}dx=\int_0^{+\infty}\frac{1}{1+p^2}dp = \arctan p\Big|_0^{+\infty}=\frac{\pi}{2}$

∴ $\int_{-\infty}^{+\infty}\frac{\sin x}{x}dx=\pi$.

法五: 我们知道 $\delta(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\cos kx\,dk$

是 $\delta(x)$ 的一个常用表达式.

令 $F(\lambda)=\int_{-\infty}^{+\infty}\frac{\sin\lambda x}{x}dx$

不加证明地给出 $F'(\lambda)=\int_{-\infty}^{+\infty}\cos\lambda x\,dx = 2\pi\delta(\lambda)$

$\eta(x)=\begin{cases}1 & x>0\\0 & x<0\end{cases}$

积分: $F(\lambda)=2\pi\eta(\lambda)+C$

不妨取 $\lambda>0$. $F(\lambda)=2\pi+C$

观察发现 $F(\lambda)$ 为奇函数.

$F(\lambda)=-F(-\lambda)$ ∴ $C=-\pi$.

即 $F(\lambda)=\begin{cases}\pi & \lambda>0\\-\pi&\lambda<0\end{cases}$

特殊地. 取 $\lambda=1$ $F(\lambda=1)=\int_{-\infty}^{+\infty}\frac{\sin x}{x}dx=\pi$

法六:

consider the fourier tansmition of $f(x)=\begin{cases}1&|x|\le1\\0&|x|>1\end{cases}$

$F(\omega)=\int_{-\infty}^{+\infty}f(x)e^{i\omega x}dx = \int_{-1}^{1}e^{i\omega x}dx=\frac{1}{i\omega}e^{i\omega x}\Big|_{-1}^{1}=\frac{e^{i\omega}-e^{-i\omega}}{i\omega}=\frac{2}{\omega}\sin\omega$

反演: $f(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}F(\omega)e^{-i\omega x}d\omega$

取 $x=0$, $f(0)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{2}{\omega}\sin\omega\, d\omega = 1$

即 $\int_{-\infty}^{+\infty}\frac{\sin\omega}{\omega}d\omega=\pi$

LG 答题策略

一、不需按顺序答题、

先把会做的、能拿到分的做了.

二、一道题完整答完 30 min 左右.

三、看准力学题系统摆放方式:

水平（光滑）放置 — 不考虑重力.

竖直放置 — 考虑重力.

四、关注题干（电）.

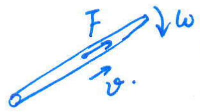
已知静电力常量 k.

or 真空介电常数 ε_0.

μ_0 — k 同理.

五、易错点.

a. 铰接杆. 径向作用力（有位移）内力作功.



$\int \vec{F}\cdot d\vec{r} \neq 0$.

b. 电像法求势能 像电荷注意.

△ c. 弹簧振子 落地跳动 平衡位置变动!

d. $\vec{D}$ 电极位移矢量不完全由 ρ_{e0} 决定.

$\vec{D} \neq \vec{D}(\vec{E}_0)$

$\vec{D} = \vec{D}(\vec{E})$

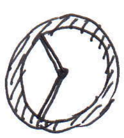
$= \varepsilon_0\vec{E} + \vec{P}$

$= \varepsilon_0\vec{E} + \chi_e\varepsilon_0\vec{E}$

$= \varepsilon\vec{E}$

$\|\varepsilon_1|\varepsilon_2|\varepsilon_3\|$ 此类情况才有 $\vec{D} = \varepsilon_0\vec{E}_0$

e. 角度制、弧度制

f. 

不均匀圆盘.

i) 平行轴只能对质心用

ii) 动能定理.

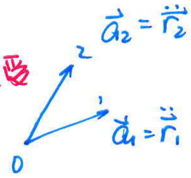
动能求法.

$E_k = \frac{1}{2}I_{瞬}\omega^2 = \frac{1}{2}Mv_c^2 + \frac{1}{2}I_c\omega^2$.

仅此两种.

iii) 角动量定理 需要补惯性力!

g. 约化质量:（内力）两体，仅受内力.



$-\vec{F} = m_1\vec{a}_1$

$+\vec{F} = m_2\vec{a}_2$

记 $\vec{a} = \vec{a}_2 - \vec{a}_1$

$\vec{a} = \left(\frac{1}{m_2} + \frac{1}{m_1}\right)\vec{F}$

$\therefore \vec{F} = \frac{m_1 m_2}{m_1+m_2}\vec{a}$ ，（$\mu = \frac{m_1 m_2}{m_1+m_2}$）.

* 约化质量等效的是惯性质量. 而内力大小不变

· 注意. 在万有引力场中答案结果含有 m 可能源于 $\frac{GMm}{r^2}$. 故不可直接换为 μ.（谨慎）

在库仑力场等与质量无关内力场中:

h. 弹性绳振动上方可能出现上抛运动.

LG 公式单.

<<力学>>

$\rho = \dfrac{v^2}{a_\perp} = \dfrac{dl}{d\theta} = \dfrac{\dot{l}}{\dot{\theta}}$ 都相同

⇒ 一定要同与 $\vec{v}\perp$ 的 a 分量 ☆

一、运动学.

0. $\vec{r}(t)$　$\vec{v}(t) = \frac{d}{dt}\vec{r}(t)$　$\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = \frac{d^2}{dt^2}\vec{r}(t)$

1. 自然坐标系：（$\hat{e}_n$ 向外为正）.

$\vec{v} = \dfrac{ds}{dt}\cdot\hat{e}_\tau$　　$\vec{a} = -\dfrac{v^2}{\rho}\hat{e}_n + \dfrac{dv}{dt}\hat{e}_\tau$

2. 平动参考系：　$= \vec{\omega}\times\vec{v} + \vec{\beta}\times\vec{R}$

$\vec{v} = \vec{v}' + \vec{v}_0$,　$\vec{a} = \vec{a}' + \vec{a}_0$　（相对 + 牵连）.

3. 转动系：

$\boxed{\vec{v} = \vec{v}' + \vec{\omega}\times\vec{r}}$

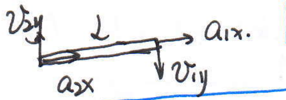
$\boxed{\vec{a} = \vec{a}' + \vec{\beta}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + 2\vec{\omega}\times\vec{v}'}$

4. 约束.

1) 绳 ： 沿绳 v 相等.（绳长不变）.

$a_{\text{t忘}} = \dfrac{(v_{1\tau} - v_{2\tau})^2}{l}$　　$v_{n1} = v_{n2}$

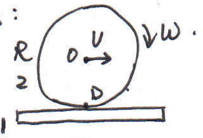
2) 杆 ： 沿杆 v 等.

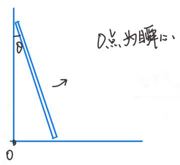


$\boxed{v_{n1} = v_{n2}}$

$\boxed{a_{2x} - a_{1x} = \dfrac{(v_{2y} + v_{1y})^2}{l}}$

3) 轮：





无滑：$v = \omega R$.

D 为瞬心.　　$\vec{v}$ 法 连续.

接触点 v 切等.

$\vec{a}_0' = \vec{\beta}\times\vec{R} + \vec{\omega}\times(\vec{\omega}\times\vec{R})$.

$\boxed{4)\ \vec{r} \text{ 约束} (\tfrac{d}{dt}) \Rightarrow \vec{v} \text{ 约束} (\tfrac{d}{dt}) \Rightarrow \vec{a} \text{ 约束}}$

二、动力学：

1. 惯性力：

1) 加速平动系.

$\vec{F} = -m\vec{a}_s$.

2). 转动系.

$$\boxed{\begin{aligned}\vec{F} &= -m\vec{\beta}\times\vec{r} - m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m\vec{\omega}\times\vec{v} \\ &= -m\vec{\beta}\times\vec{r} + m\omega^2\vec{R} - 2m\vec{\omega}\times\vec{v}\end{aligned}}$$

「补充：• 牛二定律 加速度对质心列.
• 角动量定理 选点 可以自由选取，但如果不选质心而选，需要补惯性力」

2. 守恒量.　特殊：正则角动量. L_{eff}. 守恒！

具体问题，具体分析.

动量：

$\vec{p} = \sum m_i\vec{v}_i$　　x.y.z 三组.

角动量.

$\vec{L} = \sum m_i\vec{r}_i\times\vec{v}_i$　　x.y.z 三组.

用广义动量守恒，消参 保留某一广义坐标，得 $V(r,\theta,\varphi,\cdots)$ → $V_{equ}(r)$

能量.

$E = \sum\frac{1}{2}m_iv_i^2 + \sum E_{pi}$　　1 个.

$E_k = E_{\text{对心}} + E_{\text{质心}}$

⇒ 质心动能定理.

$\Delta E_c = \sum\vec{F}_i\times\vec{r}_c$

↑ 质心位移.

Langevein

3. 变质量.

$\vec{F} = \dfrac{d\vec{p}}{dt}$.　　$v \to v + dv$.

$\vec{p} = m\vec{v}$.　　$m \to m + dm$.

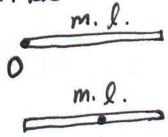
$\boxed{\vec{F} = m\dfrac{d\vec{v}}{dt} + \vec{v}\dfrac{dm}{dt}}$

4. 刚体.

$I = \sum_i m_ir_i^2$ ---- 核心公式.

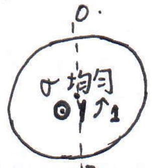
$= \iiint \underset{\frac{m}{V}}{\rho} r^2 dV = \iint \underset{\frac{m}{S}}{\sigma} r^2 dS = \int \underset{\frac{m}{l}}{\lambda} r^2 dl$.

1) 经典模型.



$\boxed{I_0 = \frac{1}{3}ml^2}$

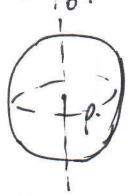
$\boxed{I_0 = \frac{1}{12}ml^2}$



圆盘：$\boxed{I_1 = \frac{1}{2}mR^2}$

△ $\boxed{I_2 = \frac{1}{4}mR^2}$

球壳：$\boxed{I_0 = \frac{2}{3}mR^2}$

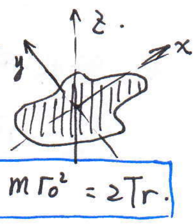


实心球：$\boxed{I_0 = \frac{2}{5}mR^2}$

2) 定理.

i) 平行轴：$\boxed{I_{MN} = I_c + md^2}$

ii) 垂直轴（平板）. $\boxed{I_x + I_y = I_z}$



iii) "迹"：$\boxed{I_x + I_y + I_z = 2\sum mr_0^2 = 2Tr.}$

iv) 自相似.

v) $I = k\rho r^n$.

"量纲"法.

⇒ $k = \cdots$

3) 刚体动能.

$$\boxed{E_k = \frac{1}{2}m v_c^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}I_{转动时瞬心}\,\omega^2}$$

转动角动量.

$$\vec{L} = m\vec{r}\times\vec{v}_c + I\vec{\omega}$$

动量.

$$\vec{p} = m\vec{v}_c$$

△ 转动惯量不一定为常量!!

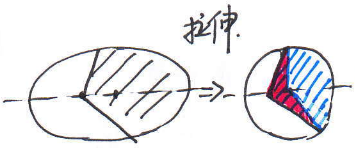
《力学》P181 瞬时轴转动定理:

$$\boxed{M_{外,M} = I_M\beta + \frac{1}{2}\omega\frac{dI_M}{dt}}$$

三、有心力、天体运动.

1、有心力. $\vec{F} = F(r)\hat{r}$.

拉伸.



2、开普勒. ① 椭轨.

② 面积速度 ⇒ 椭圆部分面积 → 三角形!!

③. $\frac{a^3}{T^2} = C$.

3、守恒 ⇒ 能量、角动量.

下落时间

i) 面积/面积速度.

ii) $t = \int_{r_1}^{r_2}\frac{dr}{\dot{r}}$

iii) $t = \int_{\theta_1}^{\theta_2}\frac{d\theta}{\dot{\theta}}$

4. $m(\ddot{r} - r\dot{\theta}^2) = F(r)$.

$r^2\dot{\theta} = h \ (= \frac{L}{m})$.

#47 GPhO! 对有心力场.(角动量守恒)! 有: $\dot{r} = -\frac{dv_\theta}{d\theta}$!

$E_k + E_p = E$. $E_p = -\int F(r)dr$.

* $E_k = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$. $V_{equ} = \frac{1}{2}\frac{L^2}{mr^2} - \frac{GMm}{r}$

$r^2\dot{\theta} = h$

$2r\dot{\theta}dr + r^2 d\dot{\theta} = 0$.

$dr = -\frac{r^2 d\dot{\theta}}{2r\dot{\theta}}$

$2\dot{\theta}dr = -rd\dot{\theta}$.

$\dot{r} = -\frac{r\,d\dot{\theta}}{2\,dt} = -\frac{d(r\dot{\theta})}{dt}$

$d(r\dot{\theta}) = rd\dot{\theta} + dr\cdot\dot{\theta} = \frac{1}{2}rd\dot{\theta}$

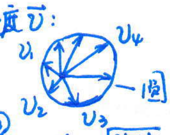
5. 比耐方程.

$$\boxed{h^2u^2\left(\frac{d^2u}{d\theta^2} + u\right) = -\frac{F(r)}{m}}$$

$$\begin{cases} u = \frac{1}{r} \\ h = \frac{L}{m} \end{cases}$$

PS: ① $\frac{a^3}{T^2} = \frac{GM}{4\pi^2}$

② 速度$\vec{v}$:



②⇒③ $v_1 = v_3 = \sqrt{v_2 v_4}$.

6. 椭圆相关.(抛;双曲).

$$\boxed{e = \frac{c}{a} \qquad p = \frac{b^2}{c} \qquad p^* = \frac{b^2}{a}}$$

离心率　　焦准距　　半通径长.

极坐标方程: $\boxed{r = \frac{ep}{1+e\cos\theta} = \frac{p^*}{1+e\cos\theta}}$

$$\boxed{p^* = \frac{L^2}{GMm^2} = \frac{(L/m)^2}{GM}}$$

$$\boxed{e = \sqrt{1 + \frac{2EL^2}{G^2M^2m^3}}}$$

*. 隆格楞次矢量.

$$\vec{R} = \vec{v}\times\vec{L} - GMm\frac{\vec{r}}{r}.$$

性质: 方向指向近日点,

大小正比于偏心率

$\vec{R}$ 为一守恒量.

$\vec{r}\cdot\vec{R}$ 可快速推极坐标方程

7. $E \sim 2a$. $L \sim p^*$.

$$\boxed{\begin{aligned} &抛: E = 0. \\ &椭: E = -\frac{GMm}{2a} < 0. \\ &双曲: E = \frac{GMm}{2a} > 0 \end{aligned}}$$

$$\boxed{L = \sqrt{GMm^2p^*}}$$

$e=0$ ~ 圆　$0<e<1$ ~ 椭圆

$e=1$ ~ 抛物线　$e>1$ ~ 双曲线

8. 约化质量.

(两者选其一,不可同时用※

(i) $F = \mu a$. $\boxed{\mu = \frac{Mm}{M+m}}$

$\frac{GMm}{r^2} = \frac{Mm}{M+m}a$.

(ii) $\boxed{F = \frac{G(M+m)m}{r^2}}$

$\Rightarrow \frac{G(M+m)m}{r^2} = ma$.

雷诺数 $Re = \rho v r/\eta$

泊肃叶公式 $Q_V = \frac{\Delta p\pi R^4}{8\eta L}$

合阻力:(球) $f = 6\pi r\eta v$

四、振动&机械波.

$x = A\cos(\omega t \pm kx + \varphi) = Re(Ae^{i(\omega t \pm kx + \varphi)})$.

左行为加. 右行为减

物理量:　　波速(相速).

ω. $\nu = f$. k. λ. u. T

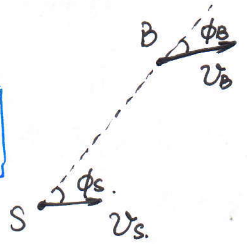
$\boxed{T = \frac{2\pi}{\omega}}$ $\nu = \frac{1}{T}$ $\boxed{\omega = 2\pi\nu}$

$\boxed{\lambda = uT = \frac{u}{\nu}}$ $\boxed{k = \frac{2\pi}{\lambda} = \frac{\omega}{u}}$

多普勒效应:

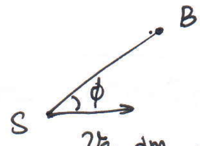
非相对论:(子收母源).

$$\boxed{\nu = \frac{u - v_B\cos\phi_B}{u - v_S\cos\phi_S}\nu_0}$$

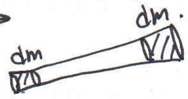


相对论下:

$$\boxed{\nu = \frac{\sqrt{1-\beta^2}}{1-\beta\cos\phi}\nu_0}$$ $\beta = \frac{v_S}{c}$.



(以B为系,B系中所夹角ϕ).



五、流体.

$\Delta W = \Delta E_k + \Delta U + \Delta E_p$.

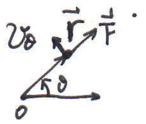
不可压缩 → ρ恒; $\frac{\partial\rho}{\partial t} = 0$

1. 连续性方程. $\boxed{\oint\rho\vec{v}\cdot d\vec{S} + \iiint\frac{\partial\rho}{\partial t}dV = 0}$ $\xrightarrow{定常}$ $\boxed{\oint\vec{v}\cdot d\vec{S} = 0}$

2. 伯努力方程. $\boxed{p + \frac{1}{2}\rho v^2 + \rho gh = const}$

↳ 推广: $\frac{dm}{\rho}(P_1 - P_2) = \frac{1}{2}dm(v_2^2 - v_1^2) + \frac{dm}{\mu}C_V(T_2 - T_1)$

<力学>. 有心场(力场)结论.



$$\vec{F} = \frac{k}{r^n} \cdot \frac{\vec{r}}{r}.$$

角动量守恒: $L = m r v_\theta$ ①

$v_\theta = \dot{\theta} r$ ②.

① ⇒ $r = \frac{L}{m} \cdot \frac{1}{v_\theta}$ ③

② ⇒ $\dot{\theta} = \frac{v_\theta}{r} = \left(\frac{L}{m}\right)^{-1} v_\theta^2$ ④

其二: $F = m(\ddot{r} - r\dot{\theta}^2) = \frac{k}{r^n}$ ⑤.

$$\dot{r} = \frac{dr}{dv_\theta} \cdot \frac{dv_\theta}{d\theta} \cdot \frac{d\theta}{dt}$$

$$= \left(\frac{dr}{dv_\theta}\right) \cdot \dot{\theta} \cdot \frac{dv_\theta}{d\theta}$$

③④ ⇒ $= -\frac{L}{m} \cdot \frac{1}{v_\theta^2} \cdot \left(\frac{L}{m}\right)^{-1} v_\theta^2 \cdot \frac{dv_\theta}{d\theta}$

$= -\frac{dv_\theta}{d\theta}$ ⑥.

⑥: $\dot{r} = -\frac{dv_\theta}{d\theta}$ 的结论仅要求有心力场.
(即角动量守恒).

$$\ddot{r} = -\frac{d}{d\theta}\left(\frac{dv_\theta}{d\theta}\right) \cdot \frac{d\theta}{dt}$$

$= -\frac{d^2 v_\theta}{d\theta^2} \cdot \left(\frac{L}{m}\right)^{-1} v_\theta^2.$ ⑦

特殊的. 对于平方反比力场. ⑤化为:

$m(\ddot{r} - r\dot{\theta}^2) = \frac{k}{r^2}$ ⑤'.

$$r^2\ddot{r} - r^3\dot{\theta}^2 = \frac{k}{m}.$$

$$-\left(\frac{L}{m}\right)^2 \cdot v_\theta^{-2} \cdot \left(\frac{L}{m}\right)^{-1} v_\theta^2 \frac{d^2 v_\theta}{d\theta^2} - \frac{L}{m} \cdot v_\theta = \frac{k}{m}.$$

$$\Rightarrow \quad \frac{L}{m} \frac{d^2 v_\theta}{d\theta^2} + \frac{L}{m} v_\theta + \frac{k}{m} = 0.$$

$$\Rightarrow \quad \frac{d^2 v_\theta}{d\theta^2} + v_\theta + \frac{k}{L} = 0.$$

可知: 令 $\check{v}_\theta = v_\theta + \frac{k}{L}$.

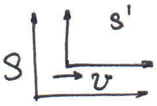
$$\frac{d^2 \check{v}_\theta}{d\theta^2} + \check{v}_\theta = 0$$

⇒ $\check{v}_\theta = A\cos\theta$ (适当的坐标系选取).

⇒ $v_\theta = A\cos\theta - \frac{k}{L}$.

LG 公式单.

<狭义相对论>



两参考系相对速度 v.

规定两系物理量:

S: $x.\ t.\ v.\ a.\ F.\ E.\ \cdots$

S': $x',\ t',\ v',\ a',\ F',\ E',\ \cdots$

公式: 记 $\beta = \frac{v}{c}$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$x = \gamma(x' + \beta c t')$ $\qquad x' = \gamma(x - \beta c t)$

$y = y'$ $\qquad y' = y$

$z = z'$ $\qquad z' = z$

$t = \gamma(t' + \frac{\beta}{c}x')$ $\qquad t' = \gamma(t - \frac{\beta}{c}x)$

*. 公式中的"±"可对照 Galileo 变换确定.

$$v_x = \frac{dx}{dt} = \frac{dx'/dt'}{dt/dt'} = \frac{v_x' + v}{1 + \frac{v_x' v}{c^2}}$$

$$v_y = \frac{dy}{dt} = \frac{dy'/dt'}{dt/dt'} = \frac{\gamma^{-1} v_y'}{1 - \frac{v_x v}{c^2}}$$

附:

$$a_x = \frac{(1-\beta^2)^{3/2}}{(1 + \frac{v u_x'}{c^2})^3} a_x'$$

$$a_y = \frac{1-\beta^2}{(1 + \frac{v u_x'}{c})^2} a_y' - \frac{(1-\beta^2)\frac{v u_y'}{c^2}}{(1 + \frac{v u_x'}{c^2})^3} a_x'$$

狭义相对论动力学:

$$\vec{E}' = \vec{E}_{\parallel} + \gamma(\vec{E}_{\perp} + \vec{v} \times \vec{B})$$

$$\vec{B}' = \vec{B}_{\parallel} + \gamma(\vec{B}_{\perp} - \frac{\vec{v}}{c^2} \times \vec{E}).$$

$$F_x' = \frac{F_x - \frac{v}{c^2}\vec{u} \cdot \vec{F}}{1 - \frac{v}{c^2}u_x}$$

$$F_y' = \frac{\sqrt{1-\beta^2}\, F_y}{1 - \frac{v}{c^2}u_x}$$

$$F_z' = \frac{\sqrt{1-\beta^2}\, F_z}{1 - \frac{v}{c^2}u_x}.$$

$P_x' = \gamma(P_x - \frac{\beta}{c}E)$ $\qquad P_x = \gamma(P_x' + \frac{\beta}{c}E)$

$P_y' = P_y$ $\qquad P_y = P_y'$

$P_z' = P_z$ $\qquad P_z = P_z'$

$E' = \gamma(E - \beta c P_x)$ $\qquad E = \gamma(E' + \beta c P_x')$.

$m = \gamma m_0$.

$E = \gamma m_0 c^2$

$P = \gamma m_0 u$.

$u = \frac{Pc^2}{E}$.

$E^2 = p^2c^2 + m_0^2c^4$.

$p = \frac{h\nu}{c}$, $E = h\nu$ 光子.

$\frac{dx}{dy} = \frac{P_x}{P_y}$.

基础知识.

1. 磁矩 $\vec{m}=I\vec{S}$ ①

$$\vec{B}=-\frac{\mu_0\vec{m}}{4\pi r_0^3}+\frac{3\mu_0\vec{r_0}(\vec{m}\cdot\vec{r_0})}{4\pi r_0^5} \quad ②$$

$$\vec{E}=-\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r_0^3}+\frac{3\vec{r_0}(\vec{p}\cdot\vec{r_0})}{4\pi\varepsilon_0 r_0^5}$$

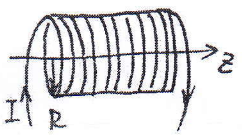
$$\varphi=\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

$\vec{L}=\vec{m}\times\vec{B}$ …均匀磁场磁偶极子所受力矩.

$\vec{F}=(\vec{m}\cdot\vec{\nabla})\vec{B}=[\vec{\nabla}(\vec{m}\cdot\vec{B})]_{\vec{m}}$ …非均匀磁场磁偶极子受力. $\vec{E}=\vec{E_0}+\vec{E'}$

$\vec{B}=\vec{B_0}+\vec{B'}$

2. 螺线管轴线上磁场强度分布



$$B_z=\frac{n\mu_0 I}{2}(\cos\beta_2-\cos\beta_1) \quad ③$$

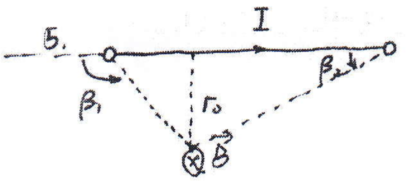
$(\mu_0 i)$

3. $\oiint_S\vec{B}\cdot d\vec{s}=0.$ ④

$\oint_L\vec{B}\cdot d\vec{l}=\mu_0\sum I.$ ⑤

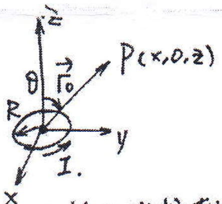
4. $d\vec{B}=\frac{\mu_0}{4\pi}\cdot\frac{Id\vec{l}\times\vec{r}}{r^3}$ ⑥

5.



$$B=\frac{\mu_0 I}{4\pi r_0}\cdot(\cos\beta_2-\cos\beta_1). \quad ⑦$$

6.



取恰当坐标轴,使P位于xOy平面.

有:

$$\begin{cases} B_x=\frac{\mu_0 IRr_0\cos\theta}{2\pi}\int_0^{\pi}\frac{\cos\varphi\,d\varphi}{(r_0^2+R^2-2Rr_0\sin\theta\cos\varphi)^{3/2}} \\ \quad=\frac{\mu_0 I}{2\pi}\cdot(\vec{R}\cdot\vec{r_0})\int_0^{\pi}\frac{\cos\varphi\,d\varphi}{(r_0^2+R^2-2Rr_0\sin\theta\cos\varphi)^{3/2}} \\ B_y=0 \\ B_z=\frac{\mu_0 IR^2}{2\pi}\int_0^{\pi}\frac{(R-r_0\sin\theta\cos\varphi)\,d\varphi}{(r_0^2+R^2-2Rr_0\sin\theta\cos\varphi)^{3/2}} \end{cases} \quad ⑧$$

当P位于z轴上. 即 $\theta=0$. $r_0=z$, 有:

$$\begin{cases} B_x=0 \\ B_y=0 \\ B_z=\frac{\mu_0 IR^2}{2(R^2+z^2)^{3/2}} \end{cases} \quad ⑨$$

当 $r_0\gg R$ 时. 化为 ②

磁化强度

$$\vec{M}=\frac{\sum\vec{m}_i\Delta V}{\Delta V} \leftrightarrow \vec{P}=\frac{\sum\vec{p}_i\Delta V}{\Delta V}$$

磁化电流 $\quad \vec{E'}=-\frac{\vec{P}}{3\varepsilon_0}$.

$\oint_L\vec{M}\cdot d\vec{l}=\sum I'$ $\quad\sim\quad$ $\oiint_S\vec{P}\cdot d\vec{s}=-\iiint_V\rho_e'\,dV$

$\vec{i'}=\vec{M}\times\vec{n}$ $\quad\sim\quad$ $\rho_e'=-\vec{\nabla}\cdot\vec{P}$

磁场强度(辅助变量).

def $\vec{H}=\frac{\vec{B}}{\mu_0}-\vec{M}$ $\quad\sim\quad$ $\vec{D}=\varepsilon_0\vec{E}+\vec{P}$

$\vec{M}=\chi_m\vec{H}$ $\quad\sim\quad$ $\vec{P}=\chi_e\varepsilon_0\vec{E}$

$\vec{B}=\mu\vec{H}$, $(\mu=\mu_0(1+\chi_m))$

$\sim\quad\vec{D}=\varepsilon_r\varepsilon_0\vec{E}=\varepsilon\vec{E}$

边值:

$\vec{n}\cdot(\vec{B_2}-\vec{B_1})=0.$ $\quad\sim\quad$ $\vec{n}\times(\vec{E_2}-\vec{E_1})=0.$

$\vec{i'}=\vec{M}\times\vec{n}=\vec{n}\times(\vec{M_2}-\vec{M_1})$ $\quad\frac{\sigma_e}{\varepsilon_0}=(\vec{E_2}-\vec{E_1})\cdot\vec{n}$

$\sim\quad\sigma_e'=(\vec{P_1}-\vec{P_2})\cdot\vec{n}$

$\vec{i_0}=\vec{n}\times(\vec{H_2}-\vec{H_1})$ $\quad\sim\quad$ $\sigma_{e0}=\vec{n}\cdot(\vec{D_2}-\vec{D_1})$

(i) 介质界面与磁感线重合 —— 界面与场线垂直.

$\vec{H}=\frac{\vec{B_0}}{\mu_0}.$ $\quad\sim\quad$ $\vec{D}=\varepsilon_0\vec{E_0}$

$\vec{B_0}$求法: $\oint\vec{B_0}\cdot d\vec{l}=\mu_0\sum I_0.$ $\quad\sim\quad$ $\oiint_S\vec{E_0}\cdot d\vec{s}=\frac{Q_0}{\varepsilon_0}$

$\vec{B_i}$求法: $\vec{B_i}=\mu_i\vec{H}=\frac{\mu_i B_0}{\mu_0}.$ $\quad\sim\quad$ $\vec{E_i}=\frac{\vec{D}}{\varepsilon_i}=\frac{\varepsilon_0\vec{E_0}}{\varepsilon_i}$

(ii) 介质界面与磁感线垂直 —— 界面与场线重合.

① 按真空处理. 解得 $\vec{B_0}$

② $\vec{B}=\alpha\vec{B_0}$ $\quad\sim\quad$ $\vec{E}=\alpha\vec{E_0}.$

α: $\oint_L\frac{\vec{B}}{\mu}\cdot d\vec{l}=\sum I_0$ $\quad\quad\alpha$:

$$\alpha=\frac{\sum I_0}{\oint_L\frac{\vec{B_0}}{\mu}\cdot d\vec{l}}$$

③ $H_i=\frac{1}{\mu_i}\vec{B}$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu \vec{H} \quad ; \mu = \mu_0 (1+\chi_m)$$

磁路定理

i 直流电路.

$$\begin{cases} \oint \vec{j} \cdot d\vec{S} = 0 . \\ \vec{j} = \sigma(\vec{E}+\vec{K}) = \sigma \vec{E}' \quad (\vec{K}: \text{非静电力}). \\ \oint \vec{K} \cdot d\vec{l} = \oint \vec{E}' \cdot d\vec{l} = \mathcal{E} \end{cases}$$

$\Rightarrow$ $\mathcal{E} = IR.$

$$R = \oint \frac{dl}{\sigma S}$$

$$R = \sum_i R_i, \quad R_i = \frac{l_i}{\sigma_i S_i} .$$

ii. 静磁场.

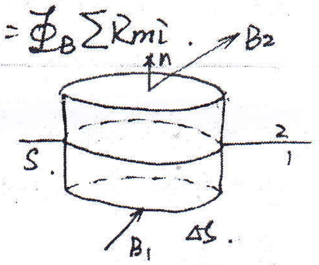
$$\begin{cases} \oint \vec{B} \cdot d\vec{S} = 0 . \\ \vec{B} = \mu \vec{H} \\ \oint \vec{H} \cdot d\vec{l} = \mathcal{E}_m = \sum I_0 . \end{cases}$$

$$R_m = \oint \frac{dl}{\mu S} .$$

$$R_m = \sum_i R_{mi} \quad , R_{mi} = \frac{l_i}{\mu_i S_i}$$

$(\Phi_B = BS \backsim I = jS.)$

$$\mathcal{E}_m = \Phi_B R_m = \Phi_B \sum R_{mi} .$$



边值关系(磁场).

$$\vec{n} \cdot (\vec{B_2} - \vec{B_1}) = 0$$

$$\vec{i}' = \vec{n} \times (\vec{M_2} - \vec{M_1})$$

$$\vec{i_0} = \vec{n} \times (\vec{H_2} - \vec{H_1}) .$$

通常, 磁介质界面 $i_0 = 0$. 故 $(\vec{H_2} - \vec{H_1}) \cdot \vec{n} = 0$

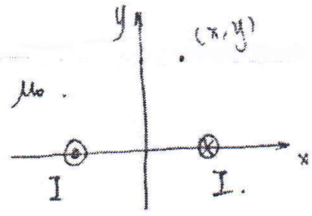
磁荷法.

$$\oiint_S \vec{H} \cdot d\vec{S} = \frac{1}{\mu_0} \sum_{(S内)} q_m .$$

$$\oint_L \vec{H} \cdot d\vec{l} = 0 .$$

$$\vec{H} = -\vec{\nabla} \varphi_m .$$

加急! 加急! 加急!!



$$B_{0x} = \frac{\mu_0 I}{2\pi} \left(\frac{-y}{(x+a)^2+y^2} + \frac{y}{(x-a)^2+y^2} \right) .$$

$$B_{0y} = \frac{\mu_0 I}{2\pi} \left(-\frac{(x-a)}{(x-a)^2+y^2} + \frac{x+a}{(x+a)^2+y^2} \right) .$$

$$\vec{J} = \frac{\sum \vec{P}_{m分子}}{\Delta V}$$

$$\oiint_S \vec{J} \cdot d\vec{S} = -\sum_{S内} q_m'$$

$$\sigma_m' = \vec{J} \cdot \vec{n}$$

$$\vec{J} = \chi_m \mu_0 \vec{H}$$

$$\vec{B} = \mu_0 \vec{H} + \vec{J}$$

$$\oiint_S \vec{B} \cdot d\vec{S} = 0 .$$

$$\vec{B} = \mu \vec{H}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_{e0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{j}_0 + \frac{\partial \vec{D}}{\partial t}$$

LG公式单.

《电学》.

一、静电场.

1. $F=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$

2. $\oint\vec{E}\cdot d\vec{S}=\frac{1}{\varepsilon_0}\sum q$

   $\oint\vec{E}\cdot d\vec{l}=0$.

3. $\varphi_P=\int_P^\infty\vec{E}\cdot d\vec{l}=\int_P^A\vec{E}\cdot d\vec{l}\ \ (\varphi_A=0)$

4. $\vec{E}=-\vec{\nabla}\varphi$

5. 定义: $\vec{P}=\frac{\sum\vec{p}_i}{\Delta V}$; $\vec{P}=\chi_e\varepsilon_0\vec{E}$

   $\vec{D}=\varepsilon_0\vec{E}+\vec{P}$

6. 关系: $\vec{E}=\vec{E}_0+\vec{E}'$

   $\vec{\nabla}\cdot\vec{P}=-\rho_e'$

   $\sigma_e'=\vec{P}\cdot\hat{n}=P_n$.

7. 电介质中的高斯定理:

   $\oint\vec{E}\cdot d\vec{S}=\frac{1}{\varepsilon_0}\sum(q_0+q')$

   $\oint\vec{D}\cdot d\vec{S}=\cancel{\frac{1}{\varepsilon_0}}\sum q_0$.

   $\Rightarrow\ \vec{\nabla}\cdot\vec{E}=\frac{1}{\varepsilon_0}(\rho_0+\rho_e')=\frac{\rho_e}{\varepsilon_0}$

   $\vec{\nabla}\cdot\vec{D}=\cancel{\frac{1}{\varepsilon_0}}\ \rho_{e0}$

8. 经典模型的电场分布:

   1) 点电荷: $\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\cdot\hat{r}$.

   q.　　$\varphi=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

   2) 无限长、均匀带电、直线.

   $\vec{E}=\frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}$

   λ.　　$\varphi=\frac{-\lambda}{2\pi\varepsilon_0}\ln r+C$.

   3) 无穷大均匀带电平面:

   $\vec{E}=\frac{\sigma_e}{2\varepsilon_0}$.

   $U\propto$ 距离差.

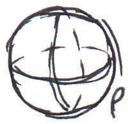
   4) 均匀带电球壳:

   $\sigma=\frac{q}{4\pi R^2}$

   $\vec{E}=\begin{cases}0 & r<R\\ \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} & r>R.\end{cases}$

   $\varphi=\begin{cases}\frac{1}{4\pi\varepsilon_0}\frac{q}{R} & r<R.\\ \frac{1}{4\pi\varepsilon_0}\frac{q}{r}. & r>R\end{cases}$

   5) 均匀带电球体.



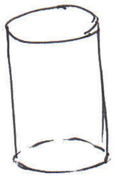
   ρ

   $\vec{E}=\begin{cases}\frac{\rho}{3\varepsilon_0}\cdot r\hat{r} & r<R\\ \frac{\rho}{3\varepsilon_0}\frac{R^3}{r^2}\cdot\hat{r} & r>R.\end{cases}$

   $\varphi=\begin{cases}\frac{\rho}{6\varepsilon_0}(3R^2-r^2) & r<R\\ \frac{\rho}{3\varepsilon_0}\frac{R^3}{r}. & r>R.\end{cases}$

   6) 均匀带电圆柱. → 外部～直导线 ～内部 $E=0$

   ＜圆筒 → 外部～直导线. ～内部 $E\propto r$, $U\propto r^2$



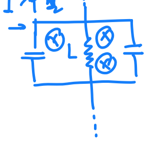
   7) 电偶极子与电偶极矩电场.

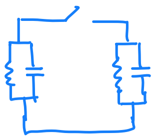
9. 有关给冲量. i 脉冲. 瞬间充电

   I 冲量

   eg.

;





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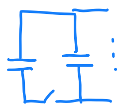


等.

   找 $\frac{d}{dt}(\square\xi-\square\eta)=0$ 守恒量.

   与 $\triangle\xi=\square\eta$ 平衡条件

   得出新的初条.

10. $\frac{L_1}{L_2}=\frac{N_1^2}{N_2^2}$　　$M_{完全}=\sqrt{L_1L_2}$

11. 关于磁介质中的磁荷观点

   $\vec{P}_m=\frac{\sum_{dV}\mu_0\vec{m}_{分}}{dV}=\mu_0\vec{M}$　　$(\sigma_m'=\vec{P}_m\cdot\vec{n},\ \rho_m'=-\vec{\nabla}\cdot\vec{P}_m)$

   $\nabla\times\vec{H}'=0$　　$\nabla\cdot\vec{H}'=\frac{\rho_m'}{\mu_0}=-\vec{\nabla}\cdot\frac{\vec{P}_m'}{\mu_0}=-\nabla\cdot\vec{M}$

   def: $\vec{B}'=\mu_0\vec{H}'+\vec{P}_m=\mu_0(\vec{H}'+\vec{M})$

   $\nabla\cdot\vec{B}'=0$.　　$\nabla\times\vec{B}'=\mu_0\nabla\times\vec{M}=\mu_0\vec{j}'$

   $\Rightarrow\ \vec{B}'\rightarrow$ 附加磁场.　　$\vec{B}'\leftrightarrow\vec{D}'=\vec{D}-\varepsilon_0\vec{E}_0$

   $\vec{H}'\leftrightarrow\vec{E}$

12.

   $\vec{E}_{内}'=-\frac{\vec{P}}{3\varepsilon_0}$

   $\vec{B}'=\frac{2\mu_0}{3}\vec{M}$

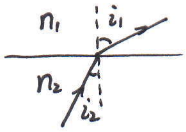
LG 公式单.

《光学》.

一、几何光学.

1. 直线传播

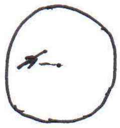
2. 折射:



$n_1\sin i_1 = n_2\sin i_2$

3. $n(r)$.

$nr\sin\theta = const$

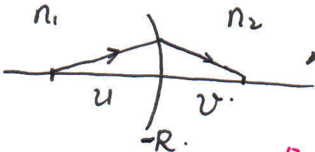


4. 成像公式:

* 正负号规定: (实像正，虚像负).

物(像)位于物(像)方空间为正，反之为负.

(1) 两介质间球面成像(傍轴条件).



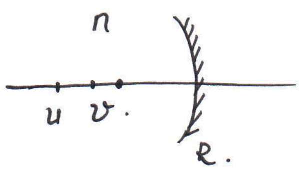
此时取$R<0$.
$-R$为半径大小.

$$\frac{n_1}{u}+\frac{n_2}{v}=\frac{n_2-n_1}{R}$$

像-物.

左负、右正、平面$R\to\infty$

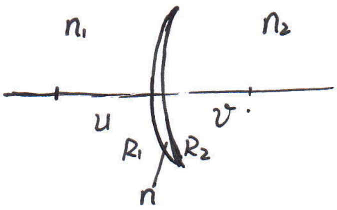
(2) 球面镜反射成像:



*: 物(像)方空间位于同一侧.
R凹向此侧为正.
反之为负，平面镜$R\to\infty$

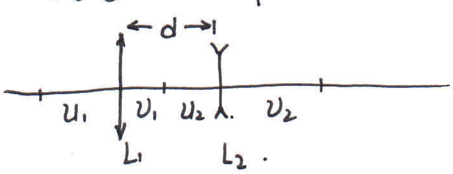
$$\frac{n}{u}+\frac{n}{v}=\frac{2n}{R}$$

(3) 通过(1)可推得:



$$\frac{n_1}{u}+\frac{n_2}{v}=\frac{n-n_1}{R_1}+\frac{n_2-n}{R_2}$$

(4) 对于多光学元件的线性成像系统:



有: $v_1+u_2=d$.

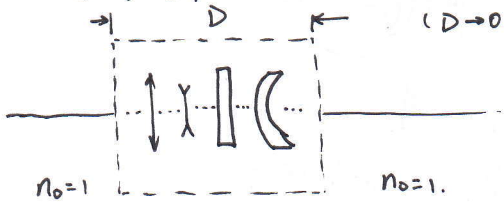
* 使用成像公式解题时，留意 $u(v)\to 0$; f; ∞ 的情况.

关于物(像)方空间的明确:

所有物(像)点的集合形成的空间称为物(像)方空间.

5. 焦距、光焦度.

1) 对于特殊成像系统:



$(D\to 0)$!!!

应有:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$ → 高斯成像公式

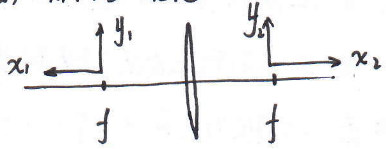
其中 $u\to\infty$时 $v\to f$. ($v\to\infty$时 $u\to f$).

f为平行光入射时焦点相对光心位置.

此时光焦度 $\Phi=\frac{1}{f}=\sum_i\Phi_i=\sum_i\frac{1}{f_i}$

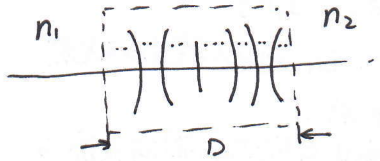
PS: 牛顿成像公式:

a) x_1、x_2 规定:



b) 有 $x_1x_2=f^2$

2) 对于一般成像系统:



$(D\to 0)$!!!

定义: 平行光入射，与光轴交点为f_2 …像方焦距.
平行光出射，与光轴交点为f_1 …物方焦距.
(f_1、f_2 的正负号法则同 u、v).

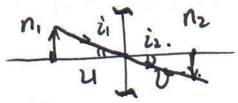
即 $u\to\infty$, $v\to f_2$; $v\to\infty$, $u\to f_1$.

应有 $\frac{f_1}{u}+\frac{f_2}{v}=1$.

同时，$\frac{n_1}{u}+\frac{n_2}{v}=\Phi=\sum\Phi_i$

△注意: 4(2)中 $\Phi=\frac{2n}{R}$ …含折射率n.

6. 横向放大率



$$\beta=-\frac{n_1 v}{n_2 u}$$

$$\beta_{总}=\prod_i\beta_i$$

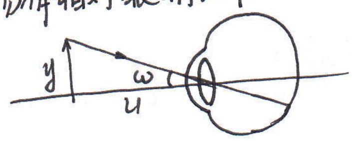
对于 $n_1=n_2$, $\beta=-\frac{v}{u}$.

由 $\frac{f_1}{f_2}=\frac{n_1}{n_2}\Rightarrow\beta=-\frac{f_1 v}{f_2 u}$ → 高斯式

又由 $\frac{f_2}{v}=\frac{u-f_1}{u}=\frac{x_1}{u}\Rightarrow\beta=-\frac{f_1}{x_1}=-\frac{x_2}{f_2}$ → 牛顿式

7. 视角：

物体相对眼睛张角.



$$\omega = \frac{y}{u}.$$

最大视角.

明视距离.

$$\omega_0 = \omega \Big|_{u=u_0=25cm}.$$

光学仪器成像在明视距离相对眼睛所张视角:

$$\omega_0' = \frac{y'}{u_0}.$$

视角放大率： $M = \frac{\omega_0'}{\omega_0}.$

八、光阑.（考纲之外）.

① 将所有光学仪器向物方空间成像.

② 相对光源（物在光轴上交点）限制最大.
（即成角最小）的称为入射光瞳（入瞳）.

③ 入瞳所对应的物称为孔径光阑（孔阑）

④ 孔阑向像方空间所成像—
出射光瞳（出瞳）.

⑤ 任意物点发出，通过入瞳中心的光线
称为主光线.

⑥ 光学系统中的所有光阑向系统的.
物方空间所成像的边缘对入射光瞳
中心的连线相对光轴所成锐角最小
时，对物面的成像范围限制最多，称
为入射窗（入窗）.

⑦ 入窗对应的共轭物为视场光阑（场阑）.

⑧ 场阑在像方空间所成像为出射窗（出窗）.

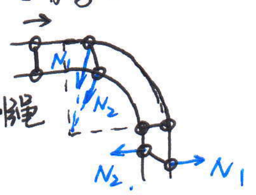
简言之：

限制 est 元件的 物 → 孔阑
共轭像（入） → 入瞳.
共轭像（出） → 出瞳

对场阑、入窗、出窗同理.

LG 补充公式单.

<力学>.



刚绳

弯轨：角动量守恒、动量不守恒

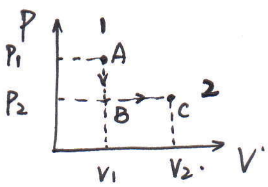
直轨：角动量不守恒 动量守恒.

水平、光滑、刚轨 [轻杆：能量不守恒 ；轻弹簧：能量守恒.

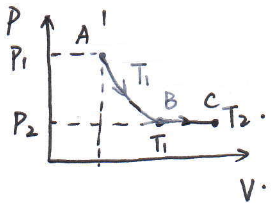
<热学>.

熵相关：对准静态过程. $dS = \frac{dQ}{T}$

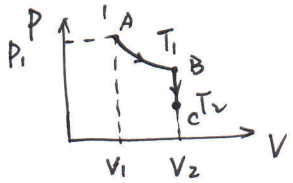
① $S(P,V)$.



② $S(P,T)$



③ $S(V,T)$.



方法：

$$dS = \left(\frac{\partial S}{\partial p}\right)_V dp + \left(\frac{\partial S}{\partial V}\right)_p dV$$

$$dS = \left(\frac{\partial S}{\partial p}\right)_T dp + \left(\frac{\partial S}{\partial T}\right)_p dT$$

$$dS = \left(\frac{\partial S}{\partial V}\right)_T dV + \left(\frac{\partial S}{\partial T}\right)_V dT.$$

分别计算 $\left(\frac{\partial S}{\partial p}\right)_V \left(\frac{\partial S}{\partial V}\right)_p \left(\frac{\partial S}{\partial p}\right)_T$

$\left(\frac{\partial S}{\partial T}\right)_p \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial S}{\partial T}\right)_V$.

过程如下：

①. $A \to B$：

$dQ = \nu C_V dT$. $\quad dS = \frac{dQ}{T} = \nu C_V \frac{dT}{T}$.

$S_B - S_A = \nu C_V \ln \frac{T_B}{T_A}$. $\quad \frac{P_1}{P_2} = \frac{T_A}{T_B}$.

$\therefore S_B - S_A = \nu C_V \ln \frac{P_2}{P_1}$.

$B \to C$.

$dQ = \nu C_p dT \quad dS = \frac{dQ}{T} = \nu C_p \frac{dT}{T}$.

$S_C - S_B = \nu C_p \ln \frac{T_C}{T_B} \quad \frac{V_1}{V_2} = \frac{T_B}{T_C}$.

$\therefore S_C - S_B = \nu C_p \ln \frac{V_2}{V_1}$.

$$\therefore \boxed{\Delta S = S_C - S_A = \nu C_V \ln \frac{P_2}{P_1} + \nu C_p \ln \frac{V_2}{V_1}}$$

② $A \to B$

$dQ = -dW = p dV$.

$dS = \frac{dQ}{T} = \frac{pdV}{T} = \nu R \frac{dV}{V}$.

$\therefore S_B - S_A = \nu R \ln \frac{V_B}{V_A} = -\nu R \ln \frac{P_2}{P_1}$.

$B \to C$.

$dQ = -dW + dU$.

$= pdV + \nu C_V dT$.

$dS = \frac{dQ}{T} = \nu R \frac{dV}{V} + \nu C_V dT$.

$\therefore S_C - S_B = \nu R \ln \frac{V_C}{V_B} + \nu C_V \ln \frac{T_C}{T_B}$

$= \nu R \ln \frac{T_2}{T_1} + \nu C_V \ln \frac{T_2}{T_1}$

$= \nu (R + C_V) \ln \frac{T_2}{T_1} = \nu C_p \ln \frac{T_2}{T_1}$

$$\boxed{\therefore \Delta S = S_C - S_A = \nu C_p \ln \frac{T_2}{T_1} - \nu R \ln \frac{P_2}{P_1}.}$$

③. $A \to B$.

$dQ = -dW = pdV$.

$dS = \frac{dQ}{T} = \frac{pdV}{T} = \nu R \frac{dV}{V}$.

$\therefore S_B - S_A = \nu R \ln \frac{V_2}{V_1}$.

$B \to C$.

$dQ = \nu C_V dT$.

$dS = \frac{dQ}{T} = \nu C_V \frac{dT}{T}$.

$\therefore S_C - S_B = \nu C_V \ln \frac{T_2}{T_1}$

$$\boxed{\therefore \Delta S = S_C - S_A = \nu R \ln \frac{V_2}{V_1} + \nu C_V \ln \frac{T_2}{T_1}.}$$

又！熵(S)为状态量与过程无关，因而
即使对非准静态过程，只要初态、末态热平衡.

即有：

$S(P,V) = \nu C_V \ln P + \nu C_p \ln V + C$.

$S(P,T) = \nu C_p \ln T - \nu R \ln P + C$.

$S(V,T) = \nu R \ln V + \nu C_V \ln T + C$.

LG终级公式单.

相对论.

$$\vec{E}' = \vec{E}_{\parallel} + \gamma(\vec{E}_{\perp} + \vec{v}\times\vec{B})$$

$$\vec{B}' = \vec{B}_{\parallel} + \gamma(\vec{B}_{\perp} - \frac{\vec{v}}{c^2}\times\vec{E})$$

$$u = \frac{Pc^2}{E}$$

$$\frac{dy}{dx} = \frac{P_y}{P_x}.$$

天体：

$$h^2u^2\left(\frac{d^2u}{d\theta^2}+u\right) = -\frac{F(r)}{m} \quad \left(u=\frac{1}{r},\ h=\frac{L}{m}\right)$$

$$L = \sqrt{GMm^2p^*}$$

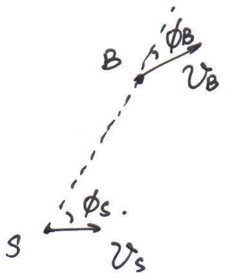
$$e = \sqrt{1+\frac{2EL^2}{G^2M^2m^3}}$$

（推荐角动量、能量守恒）.

浸渐不变量：

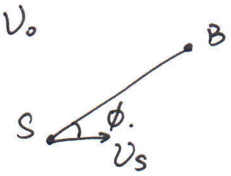
$$\oint p\,dq = \text{const}$$

多普勒效应：



（非相）$\nu = \dfrac{u-u_B\cos\phi_B}{u-v_S\cos\phi_S}\nu_0$

（相）$\nu = \dfrac{\sqrt{1-\beta^2}}{1-\beta\cos\phi}\nu_0$



均匀带电球：

$$W_自 = \frac{3}{5}\cdot\frac{q^2}{4\pi\varepsilon_0 R}$$

偶极矩：

$$\vec{E} = -\frac{\vec{p}}{4\pi\varepsilon_0 r^3} + \frac{3\vec{r}(\vec{p}\cdot\vec{r})}{4\pi\varepsilon_0 r^5}$$

$$W_p = -\vec{p}\cdot\vec{E}.$$

$$\vec{M} = \vec{p}\times\vec{E}$$

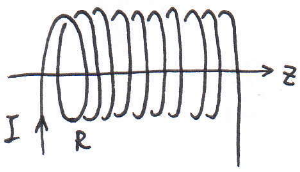
$$\vec{F} = (\vec{m}\cdot\nabla)\vec{B}$$

有自(互)感的线圈.

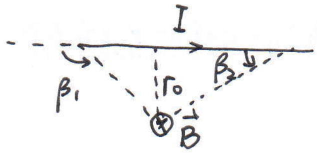
$$W_m = \vec{m}\cdot\vec{B}$$

无自(互)感的微观粒子

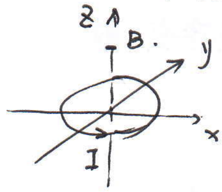
$$W_m' = -\vec{m}\cdot\vec{B}$$



$$B_z = \frac{n\mu_0 I}{2}(\cos\beta_2 - \cos\beta_1).$$



$$B = \frac{\mu_0 I}{4\pi r_0}(\cos\beta_2 - \cos\beta_1).$$



$$B_z = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$$

热

$$dU = -p\,dV + T\,dS.$$

$$f(v) = 4\pi r^2\left(\frac{m}{2\pi kT}\right)^{3/2}\exp\left(-\frac{mv^2}{2kT}\right)$$

$$v_p = \sqrt{\frac{2kT}{m}} \quad \langle v\rangle = \sqrt{\frac{8kT}{\pi m}} \quad \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}.$$

电磁能（虚功）.

$\vec{F} = -(\nabla W_e)_Q.$

$\vec{F} = (\nabla W_e)_U$

· 电：$F_x = \left(\dfrac{\partial W_e}{\partial x}\right)_U$

Tips：$F_x = -\left(\dfrac{\partial W_e}{\partial x}\right)_Q = -\left[\dfrac{\partial}{\partial x}\left(\dfrac{Q^2}{2C}\right)\right]_Q$

$$= \frac{Q^2}{2C^2}\frac{dC}{dx}$$

$$F_x = \left(\frac{\partial W_e}{\partial x}\right)_U = \left[\frac{\partial}{\partial x}\left(\frac{CU^2}{2}\right)\right]_U$$

$$= \frac{U^2}{2}\frac{dC}{dx}.$$

$L_\theta = -\left(\dfrac{\partial W_e}{\partial\theta}\right)_Q \qquad L_\theta = \left(\dfrac{\partial W_e}{\partial\theta}\right)_U. \qquad L \to 力矩$

· 磁：$F_x = \left(\dfrac{\partial W_m}{\partial x}\right)_I. \qquad \vec{F} = (\nabla W_m)_I.$

$F_x = -\left(\dfrac{\partial W_m}{\partial x}\right)_{互} \qquad \vec{F} = -(\nabla W_m)_{互}$

$L_\theta = \left(\dfrac{\partial W_m}{\partial\theta}\right)_I \qquad L_\theta = -\left(\dfrac{\partial W_m}{\partial\theta}\right)_{互}.$

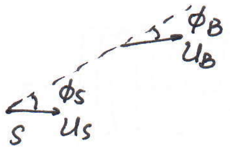
$\vec{F} = [\nabla(\vec{m}\cdot\vec{B})]_{\vec{m}} = (\vec{m}\cdot\nabla)\vec{B}.$

$\vec{F} = -(\nabla W_m')_{\vec{m}} \qquad L_\theta = -\left(\dfrac{\partial W_m'}{\partial\theta}\right)_{\vec{m}}.$

↑ 微观粒子的势能.

多普勒：

非相：$\nu = \dfrac{c-u_B\cos\phi_B}{c-u_S\cos\phi_S}$



对于 ϕ_B.ϕ_S 的说明：

ϕ_S：光传出时，光的传播方向与光源速度夹角

ϕ_B：光接收时，光的传播方向与接收者速度夹角.

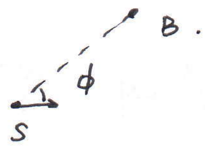
对于光的传播方向：

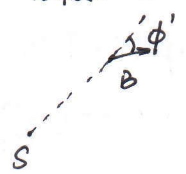
假设光（有效传输的那一束）在 $\{t_1, x_1, y_1, z_1\}$ 传出，在 $\{t_2, x_2, y_2, z_2\}$ 被接收，则

$$\hat{k} = \frac{(x_2-x_1,\ y_2-y_1,\ z_2-z_1)}{|(x_2-x_1,\ y_2-y_1,\ z_2-z_1)|}$$

相：$\nu = \dfrac{\sqrt{1-\beta^2}}{1-\beta\cos\phi} = \dfrac{1-\beta\cos\phi'}{\sqrt{1-\beta^2}}$

接收者系：S 光源系：S'





1. 数列递推技巧.

1) 形如:　(数学含量99%).

$aI_{n+2}+bI_{n+1}+cI_n=D$ 类.

令 $L_n=I_{n+1}+d$. 使:

$aL_{n+2}+bL_{n+1}+cL_n=0$

计算特征根:

$a\lambda^2+b\lambda+c=0$.

解得:

$\Rightarrow \lambda_1.\ \lambda_2$.

则 $L_n=C_1\lambda_1^n+C_2\lambda_2^n$

$\Rightarrow I_{n+1}=L_n-d$.

2) 形如:

$$R_n=\frac{CR_{n-1}+D}{AR_{n-1}+B}$$ 类.

[一] 求不动点.

$$x=\frac{Cx+D}{Ax+B}.$$

$\Rightarrow Ax^2+(B-C)x-D=0$.

① $\Delta\neq0$. $x_1\neq x_2$.

可构造数列 $\{A_n\}$

其中. $A_n=\dfrac{R_n-x_1}{R_n-x_2}$

$A_0=\dfrac{R_0-x_1}{R_0-x_2}$

则可证 $A_n=q A_{n-1}$

$$q=\frac{Ax_1-C}{Ax_2-C}.$$

$$\frac{R_n-x_1}{R_n-x_2}=\frac{R_0-x_1}{R_0-x_2}q^n$$

② $\Delta=0$. $x_1=x_2=x$.

构造等差数列 $\{A_n\}$

$A_n=\dfrac{1}{R_n-x}$, $A_0=\dfrac{1}{R_0-x}$

$A_n=A_{n-1}+d$.

$$\frac{1}{a_n-x}=\frac{1}{a_0-x}+n\left(\frac{A}{C-Ax}\right)$$

LG 公式单.

《热学》

$$\langle v_x \rangle = \frac14 \langle v\rangle$$

$\downarrow$

对 $v_x>0$ 部分平均, $=\frac12\langle|v_x|\rangle$

$$\langle \mathrm{ReLU}(v_x)\rangle = \frac14\langle v\rangle$$

$$\eta = \frac13\rho\bar v\lambda$$

$$\kappa = \frac13 \rho\bar v\lambda c_v$$

$$D=\frac13\bar v\lambda$$

$$f(\lambda)=\frac{1}{\bar\lambda}\exp\left(-\frac{\lambda}{\bar\lambda}\right).$$

$$v_p=\sqrt{\frac{2kT}{m}} \qquad \langle v\rangle=\sqrt{\frac{8kT}{\pi m}}$$

$$\sqrt{\langle v^2\rangle}=\sqrt{\frac{3kT}{m}}$$

$$\ddot{\vec x}+\gamma\dot{\vec x}=\vec f(t)$$

$$\gamma=\frac{6\pi\eta a}{m}$$

$$\vec x\cdot\ddot{\vec x}+\gamma\vec x\cdot\dot{\vec x}=\vec x\cdot\vec f(t)$$

$$\frac12\frac{d^2}{dt^2}(\vec x^2)-\dot{\vec x}^2+\frac12\gamma\frac{d}{dt}(\vec x^2)=\vec x\cdot\vec f(t)$$

期望: $\frac{d^2}{dt^2}(\vec x^2)+\gamma\frac{d}{dt}(\vec x^2)=2\langle\dot{\vec x}^2\rangle$

$=2\times\frac{3kT}{m}$

$$\overline{\vec x^2(t)}=Ae^{-\gamma t}+\frac{6kT}{\gamma m}t+C$$

$t\to\infty \Rightarrow$

$\overline{\vec x^2(t)}=2Dt+C$. 合理选 t_0 可视 $C=0$

$$2D=\frac{6kT}{6\pi\eta a}$$

$$\therefore D=\frac{kT}{2\pi\eta a}$$

$$f(v_x)=\sqrt{\frac{m}{2\pi kT}}\exp\left(-\frac{mv_x^2}{2kT}\right)$$

$$f_3(v)=4\pi v^2\left(\frac{m}{2\pi kT}\right)^{3/2}\exp\left(-\frac{mv^2}{2kT}\right)$$

$$f(p)=A\exp(-\beta p)\,dp.$$

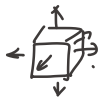
$$\beta=\frac{c}{kT}.$$

$$\int_0^{+\infty}p^n\exp(-\beta p)\,dp$$

$$=\left(-\frac{\partial}{\partial\beta}\right)^n\left(\frac1\beta\right)$$

$$f(\vec x)=A\exp\left(-\frac{\varepsilon}{kT}\right) \qquad \varepsilon=T+V(\vec x).$$

光子气:



$\frac16 nc\cdot 2p$

$$\begin{cases} p=\frac13u \\ \left(\frac{\partial u}{\partial V}\right)_T=T\left(\frac{\partial S}{\partial V}\right)_T-p \end{cases} \qquad \left(\frac{\partial p}{\partial T}\right)_S.$$

$$3p=u=T\frac{dp}{dT}-p.$$

$$4p=T\frac{dp}{dT} \Rightarrow p=\sigma T^4$$

$$u=4\sigma T^4$$

$$p=\frac43\sigma T^4$$

$$J=\sigma T^4$$

$p=nkT \Rightarrow$ {光子气体. / 理想气体}

能均分: $\bar\varepsilon=\frac12(n+r+2s)kT$

三维光子: $\bar\varepsilon=3kT$

$p=\frac23 n\bar\varepsilon$ 理想气体.

$p=\frac13 n\bar\varepsilon$ 光子气体.

$$\langle v_1-v_2\rangle=\sqrt{\frac{8kT}{\pi\mu}}$$

$$\mu=\frac{m_1m_2}{m_1+m_2}. \qquad \bar\lambda=\frac{1}{\sum_i\frac{1}{\lambda_i}}$$

$$\boxed{\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x}=-1 \quad \text{两边推导}}$$

$$\begin{cases} dx = \frac{\partial x}{\partial y}dy + \frac{\partial x}{\partial z}dz \\ dy = \frac{\partial y}{\partial x}dx + \frac{\partial y}{\partial z}dz \\ dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy \end{cases}$$

$$1\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial x}{\partial y} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial x} & 0 & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} & 0 \end{bmatrix}\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}$$

本征向量.

$$\det\begin{bmatrix} -1 & \frac{\partial x}{\partial y} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial x} & -1 & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} & -1 \end{bmatrix} = 0$$

$$-1 + \frac{\partial y}{\partial x}\frac{\partial z}{\partial y}\frac{\partial x}{\partial z} + \frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x} + \frac{\partial x}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial y}{\partial z}\frac{\partial z}{\partial y} + \frac{\partial x}{\partial z}\frac{\partial z}{\partial x} = 0$$

又 $\det\begin{bmatrix} -1 & \frac{\partial x}{\partial y} \\ \frac{\partial y}{\partial x} & -1 \end{bmatrix} = 0 \Rightarrow \frac{\partial x}{\partial y}\frac{\partial y}{\partial z} = 1$

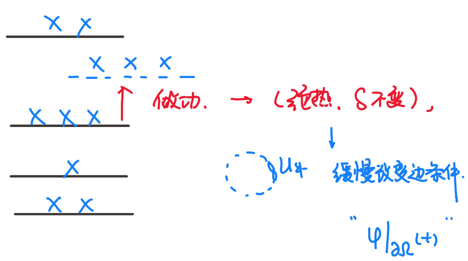
$$\Rightarrow 2\left(\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x}\right) + 2 = 0$$

$$\Rightarrow \frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x} = -1$$

做功：改变能级.

传热：改变能级分布.

书架模型： $S = k\ln\Omega$



改变分布 → TdS 变熵.

热一： $dU = đQ + đW$. $(đA = -đW)$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

$$C_p = \left(\frac{\partial H}{\partial T}\right)_p = T\left(\frac{\partial S}{\partial T}\right)_p$$

焦汤系数： $\mu = \left(\frac{\partial T}{\partial p}\right)_H$

理想气体： $\begin{cases} pV = \nu RT \\ U = U(T). \end{cases}$ 可以推出. $\left(\frac{\partial U}{\partial V}\right)_T = 0$

$C_{p,m} - C_{V,m} = R$ (迈耶公式).

↳ 成立条件：理想气体.

更广泛的： $C_{p,m} - C_{V,m} = T\left(\frac{\partial S}{\partial T}\right)_p - T\left(\frac{\partial S}{\partial T}\right)_V$

$\left(\frac{\partial S}{\partial T}\right)_p = \left(\frac{\partial S}{\partial T}\right)_V + \left(\frac{\partial S}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_p \qquad S = S(T, V(T,p))$

$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$

故 $C_{p,m} - C_{V,m} = T\left(\frac{\partial p}{\partial T}\right)_V\left(\frac{\partial V}{\partial T}\right)_p$

$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$

(可逆)卡诺热机.

$\eta_{冷} = \frac{Q_{吸}}{A}$

热二： $dS \geq \frac{dQ}{T}$

准静态、取等 $= 0$ (理气). $= \frac{a}{V^2}$ (范).

由卡诺定理 ⇒

· $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$

· $u = \alpha - T\frac{d\alpha}{dT}$ 液体表面张力.

↳ u：单位面积表面内能.
α：表面张力系数.

$dU = TdS + \alpha dA$

$u = \left(\frac{\partial U}{\partial A}\right)_T = \alpha + T\left(\frac{\partial S}{\partial A}\right)_T$

$= \alpha - T\left(\frac{\partial \alpha}{\partial T}\right)_A = \alpha - T\frac{d\alpha}{dT}$

$dF = -SdT + \sigma dA \qquad \left(\frac{\partial F}{\partial T}\right)_A = -S \quad \left(\frac{\partial F}{\partial A}\right)_T = \sigma$

由 $A=0$ 时. $F=0$ $S=0$ 推得：

$F = \int_0^T -S\,dT + \int_0^A \sigma(T)\,dA + F(T=0, A=0)$

$= \sigma(T)A$

$S = k\ln W.$

晶体结合能：

$\frac{A_n}{r^n} = \frac{\alpha e_1 e_2}{4\pi\varepsilon_0 r}$

$E_p = \frac{A_m}{r^m} - \frac{A_n}{r^n} \qquad m > n$

※ 固体热容.

· 杜隆－珀蒂定律. 认为每一粒子3个振动自由度.

$\Rightarrow U_m = 3RT \rightarrow C_m = \frac{dU_m}{dT} = 3R.$

只有充分高的温度下与实验符合. (有部分不符如金刚石)
低温失效.

· 爱因斯坦模型： 一维谐振子振动 ⇔ 晶格振动

$E_n = \hbar\omega(n+\frac{1}{2}) \qquad a_n \propto e^{-\beta\hbar\omega(n+\frac{1}{2})}$

$U_{1D} = \frac{\sum a_n E_n}{\sum a_n} = \hbar\omega\left(\frac{1}{e^{\beta\hbar\omega}-1} + \frac{1}{2}\right)$

$U_{3D} = 3\hbar\omega\left(\frac{1}{e^{\beta\hbar\omega}-1} + \frac{1}{2}\right) \qquad c = \frac{\partial U}{\partial T} = 3k(\beta\hbar\omega)^2\frac{e^{\beta\hbar\omega}}{(e^{\beta\hbar\omega}-1)^2}$

爱因斯坦模型 低温有偏差.

• 德拜模型:

固体振动 ⇒ 集体运动. 平面波.

$$U = \frac{3\pi\hbar L^3}{2\pi^2(\beta\hbar)^4 v^3}\underbrace{\int_0^\infty dx\,\frac{x^3}{e^x-1}}_{\frac{\pi^4}{15}}$$

$$c = \frac{\partial U}{\partial T} \propto T^3$$

低温区与实验一致, 但高温区 $c \xrightarrow{T\to\infty} \infty$

德拜认为应有频率上限 ω_D. → 高温区 ⇒ 杜模型

但是在极低温时. $c \xrightarrow{T\to 0} \underline{a}T + bT^3$

自由电子气的贡献

空位数:

$$n' = N e^{-u'/kT}$$ u':粒子从内→表面耗能

填隙数:

$$n'' = N e^{-u''/kT}$$ u'':粒子从表面→隙耗能.

晶体中扩散:

$$dn = -D\,\frac{dc}{dz}\,ds\,dt$$

浓度. 扩散系数.

扩散激活能

$$D = D_0 \exp\left(-\frac{Q}{RT}\right)$$

液体.

压缩性质:

$$K = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

$K^{(L)} \sim 10^{-5}\ atm^{-1}$ (liquid)

$K^{(S)} \sim 10^{-6}\ atm^{-1}$ (solid)

热膨胀:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$

$\alpha^{(H_2O)} \sim 10^{-4}\ K^{-1}$

$\alpha^{(Cu)} \sim 10^{-3}\ K^{-1}$

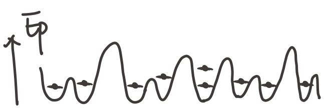
H_2O (0°C−4°C) 反常膨胀.

热容: 杜隆-珀替 → $C_{p,m}^{(L)} \approx 3R$

一般 T↑ c_p↑, 也有 H_2O (−35°C) 反常.

$$C_p - C_v = VT\frac{\alpha^2}{K}$$

热运动. 定居时间:



$$\bar{\tau} = \bar{\tau}_0\, e^{E_a/kT}$$ 势垒深度平均值. E_a.

$$\eta = \eta_0\, e^{E_a/kT}$$

$$D \propto \frac{1}{\tau} \Rightarrow D = D_0\, e^{-E_a/kT}$$

$$D_0 = \frac{1}{6}\frac{\delta^2}{\tau}$$

$$\sigma = (1-\xi)\frac{\Lambda_m}{N_A^{1/3}}\left(\frac{\rho}{\mu}\right)^{2/3}.$$

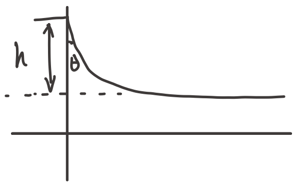
内部:表面键能. "$N_A d^3 \rho = \mu$"

等温条件下做功 ⇒ 自由能变化.

$$dF = dW - SdT \qquad \text{"}dT=0\text{"}$$

表面内外压强差:

$$\Delta p = p_i - p_o = \sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$



已知. ρ σ. 浸润角 θ

$$h = \sqrt{\frac{2\sigma}{\rho g}(1-\sin\theta)}$$

此处仅列出微分方程: $\rho g y = \frac{\sigma}{2}\cdot\frac{y''}{[1+y'^2]^{3/2}}$

化学势: $\mu = G/N$.

开放系统热力学关系:

$$\begin{cases} dU = TdS - pdV + \mu dN \\ dH = TdS + Vdp + \mu dN \\ dF = -SdT - pdV + \mu dN \\ dG = -SdT + Vdp + \mu dN \end{cases}$$

单元系复相平衡:

$$\delta U = \delta U_\alpha + \delta U_\beta = 0$$
$$\delta V = \delta V_\alpha + \delta V_\beta = 0$$
$$\delta N = \delta N_\alpha + \delta N_\beta = 0$$

$$\delta S = \left(\frac{1}{T_\alpha}-\frac{1}{T_\beta}\right)\delta U_\alpha + \left(\frac{p_\alpha}{T_\alpha}-\frac{p_\beta}{T_\beta}\right)\delta V_\alpha - \left(\frac{\mu_\alpha}{T_\alpha}-\frac{\mu_\beta}{T_\beta}\right)\delta N_\alpha = 0$$

克拉珀龙方程:

$$\mu_\alpha = \mu_\beta. \qquad d\mu_\alpha = d\mu_\beta$$

$$-S_\alpha dT_\alpha + V_\alpha dp_\alpha = -S_\beta dT_\beta + V_\beta dp_\beta$$

$$(S_\beta - S_\alpha)dT = (V_\beta - V_\alpha)dp$$

$$\frac{dp}{dT} = \frac{T(S_\beta - S_\alpha)}{T(V_\beta - V_\alpha)} = \frac{\Lambda_m}{T(V_\beta - V_\alpha)}.$$

二级相变: $S_\alpha = S_\beta$, $V_\alpha = V_\beta$ 的情况:

"洛必达"

$$\frac{dp}{dT} = \frac{\left(\frac{\partial S_\beta}{\partial p}\right)_T - \left(\frac{\partial S_\alpha}{\partial p}\right)_T}{\left(\frac{\partial V_\beta}{\partial p}\right)_T - \left(\frac{\partial V_\alpha}{\partial p}\right)_T} = \frac{\left(\frac{\partial V_\beta}{\partial T}\right)_S - \left(\frac{\partial V_\alpha}{\partial T}\right)_S}{\left(\frac{\partial V_\beta}{\partial p}\right)_T - \left(\frac{\partial V_\alpha}{\partial p}\right)_T}$$

$$= \frac{\alpha_\beta - \alpha_\alpha}{K_\beta - K_\alpha}$$

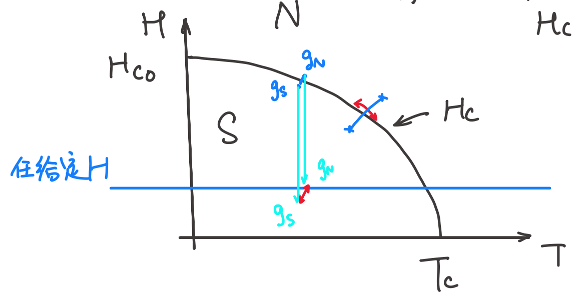
(α:压强系数 β:膨胀系数 K:压缩系数)

$$= \frac{\left(\frac{\partial S_\beta}{\partial T}\right)_p - \left(\frac{\partial S_\alpha}{\partial T}\right)_p}{\left(\frac{\partial V_\beta}{\partial T}\right)_p - \left(\frac{\partial V_\alpha}{\partial T}\right)_p} = \frac{1}{V_m T}\,\frac{C_{p\beta} - C_{p\alpha}}{\alpha_\beta - \alpha_\alpha}$$

$$\Rightarrow \Delta C_p = T V_m \frac{(\Delta\alpha)^2}{\Delta K_T}$$

超导相变.

由实验观测: $H_c = H_{c0}\left(1-\left(\frac{T}{T_c}\right)^2\right)$



S: 超导相. N: 常规相 H_c: 临界磁场

T_c. 临界温度. $g=\frac{G}{N}$ 化学势

$$du = Tds - pdV + \mu_0 V_m \vec{H}\cdot d\vec{M}$$

$$dg = sdT - \mu_0 V_m \vec{M}\cdot d\vec{H} \quad (定压)$$

我们声称: 在超导相变时 化学势连续. (trivial)

对于N相. $\vec{B}=\mu_0(\vec{H}+\vec{M})$, $\vec{M}\approx 0$

对于 S相. $\vec{B}=0$, $\vec{H}=-\vec{M}$

$$g_s(p,T,H_c) = g_N(p,T,H_c)$$

考虑随意取定的磁场大小H.

$$g_N(p,T,H_c) - g_N(p,T,H) = \int -\mu_0 V_m \overset{0}{\vec{M}}\cdot d\vec{H} = 0$$

$$g_s(p,T,H_c) - g_s(p,T,H) = \mu_0 V_m \int \vec{H}\cdot d\vec{H} = \frac{1}{2}\mu_0 V_m (H_c^2 - H^2)$$

$$\Rightarrow g_s(p,T,H) - g_N(p,T,H) = -\frac{1}{2}\mu_0 V_m (H_c^2 - H^2)$$

由于 $S = -\left(\frac{\partial g}{\partial T}\right)_{p,H}$. 由于H任取. 取 $H=H_c$.

$$\Rightarrow \boxed{S_s - S_N = \mu_0 V_m H_c \frac{dH_c}{dT}}$$

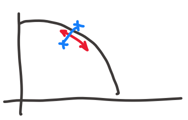
or 法二 (克拉珀龙方程)

$$g_s = g_N. \to dg_s = dg_N$$

$$-S_s dT - \mu_0 V_m \vec{M}_s\cdot d\vec{H} = -S_N dT - \mu_0 V_m \vec{M}_N\cdot d\vec{H}$$

$$\Rightarrow \frac{dH}{dT} = -\frac{S_s - S_c}{\mu_0 V_m (M_s - M_N)} \qquad \begin{cases} M_N \approx 0 \\ M_s = -H_c \end{cases}$$

$$\Rightarrow \boxed{S_s - S_c = \mu_0 V_m H_c \frac{dH_c}{dT_c}}$$



考虑界面过程, 实则算出两侧之差

特殊地. 当 $T=T_c$ 时, $H_c=0$.

$$S_s - S_N = 0 \quad \to 一阶为0.$$

故 发生二级相变.

此时. $C_{p,s} - C_{p,N} = \left(\frac{\partial S_{p,s}}{\partial T}\right)_{p,H} - \left(\frac{\partial S_{p,N}}{\partial T}\right)_{p,H}$

$$= \frac{\partial}{\partial T}\left(\mu_0 V_m H_c \frac{dH_c}{dT}\right)$$

$$= \mu_0 V_m \left(\left(\frac{dH_c}{dT}\right)^2 + H_c \frac{d^2H_c}{dT^2}\right)$$

表面张力系数的推导:

- 内部: N个键.
- 表面: ζN个键.

$W_{内\to表} = (1-\zeta)\frac{N}{2}\cdot S$.   S: 键能.

$$\alpha = \frac{dW}{dA} = (1-\zeta)\cdot\frac{N}{2}\cdot S\cdot n$$

$$n = \frac{1}{d^2},$$

汽化热:

$$\Lambda_m = \frac{N}{2}\cdot S\cdot N_A$$

$$\alpha =$$

$$N_A\cdot\rho\cdot d^3 = \mu.$$

由(3.4.16)式：

$$\delta T\delta S-\delta p\delta v>0 \quad ①$$

选(T,p)为独立变量：

$$\delta S=\left(\frac{\partial S}{\partial T}\right)_p\delta T+\left(\frac{\partial S}{\partial p}\right)_T\delta p \quad ②$$

$$\delta v=\left(\frac{\partial v}{\partial T}\right)_p\delta T+\left(\frac{\partial v}{\partial p}\right)_T\delta p \quad ③$$

由麦克斯韦关系：

$$\left(\frac{\partial S}{\partial p}\right)_T=-\left(\frac{\partial v}{\partial T}\right)_p \quad ④$$

②③④代入①式得：

$$\left(\frac{\partial S}{\partial T}\right)_p(\delta T)^2+\left(\frac{\partial S}{\partial p}\right)_T\delta p\delta T-\left(\frac{\partial v}{\partial T}\right)_p\delta T\delta p-\left(\frac{\partial v}{\partial p}\right)_T(\delta p)^2>0$$

$$\Rightarrow \left(\frac{\partial S}{\partial T}\right)_p(\delta T)^2-2\left(\frac{\partial v}{\partial T}\right)_p\delta T\delta p-\left(\frac{\partial v}{\partial p}\right)_T(\delta p)^2>0 \quad ⑤$$

由⑤式看出，由于选择(T,p)为独立变量，"$\delta T\delta p$"而交叉项并未相消，δT，δp可独立改变。

∴为使⑤成立：

$$\begin{cases}\left(\frac{\partial S}{\partial T}\right)_p>0 & ⑥\\ -\left(\frac{\partial v}{\partial p}\right)_T>0 & ⑦\\ \Delta=4\left[\left(\frac{\partial v}{\partial T}\right)_p\right]^2-4\left(\frac{\partial S}{\partial T}\right)_p\left[-\left(\frac{\partial v}{\partial p}\right)_T\right]<0 & ⑧\end{cases}$$

其中⑥、⑦分别对应：

$$C_p=T\left(\frac{\partial S}{\partial T}\right)_p>0 \quad ⑨$$

$$\kappa_T=-\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T>0 \quad ⑩$$

下面对⑧变形。

$$\left[\left(\frac{\partial v}{\partial T}\right)_p\right]^2<-\left(\frac{\partial S}{\partial T}\right)_p\left(\frac{\partial v}{\partial p}\right)_T \quad ⑧'$$

又应有：

$$-\left(\frac{\partial S}{\partial T}\right)_p\left(\frac{\partial v}{\partial p}\right)_T$$

$$=-\frac{\partial(S,p)}{\partial(T,p)}\cdot\frac{\partial(v,T)}{\partial(p,T)}$$

$$=\frac{\partial(S,p)}{\partial(p,v)}\cdot\frac{\partial(v,T)}{\partial(T,S)}\cdot\frac{\partial(p,v)}{\partial(p,T)}\cdot\frac{\partial(p,v)}{\partial(p,T)}\cdot\frac{\partial(T,S)}{\partial(p,v)}$$

$$=-\left(\frac{\partial S}{\partial v}\right)_p\cdot\left(-\left(\frac{\partial v}{\partial S}\right)_T\right)\cdot\left(\frac{\partial v}{\partial T}\right)_p\cdot\left(\frac{\partial v}{\partial T}\right)_p\cdot 1.$$

$$=\left(\frac{\partial S}{\partial v}\right)_p\left(\frac{\partial v}{\partial S}\right)_T\cdot\left[\left(\frac{\partial v}{\partial T}\right)_p\right]^2 \quad ⑨$$

结合⑧⑨ ⇒

$$\left[\left(\frac{\partial v}{\partial T}\right)_p\right]^2<\left(\frac{\partial S}{\partial v}\right)_p\left(\frac{\partial v}{\partial S}\right)_T\cdot\left[\left(\frac{\partial v}{\partial T}\right)_p\right]^2 \quad ⑩$$

由于 $\left[\left(\frac{\partial v}{\partial T}\right)_p\right]^2>0$. 两侧相约：

$$\left(\frac{\partial S}{\partial v}\right)_p\left(\frac{\partial v}{\partial S}\right)_T>1. \quad ⑪$$

又根据麦克斯韦关系：

$$\left.\begin{aligned}\left(\frac{\partial S}{\partial v}\right)_p&=\left(\frac{\partial p}{\partial T}\right)_S\\ \left(\frac{\partial v}{\partial S}\right)_T&=\left(\frac{\partial T}{\partial p}\right)_v\end{aligned}\right\} \quad ⑫$$

⑫代入⑪ ⇒

$$\left(\frac{\partial p}{\partial T}\right)_S\cdot\left(\frac{\partial T}{\partial p}\right)_v>1. \quad ⑬$$

又∵压强系数定义为：

$$\beta_v=\frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_v\ ;\ \beta_S=\frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_S.$$

代入⑬即得：

$$\frac{\left(\frac{\partial p}{\partial T}\right)_S}{\left(\frac{\partial p}{\partial T}\right)_v}=\frac{\beta_S}{\beta_v}>1 \quad ⑭$$

查阅相关资料：发现膨胀系数($\alpha=\frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$)和等温压缩系数($\kappa_T=-\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$)普遍大于0.

由 $$\beta_v=\frac{\alpha}{p\kappa_T} \quad ⑮$$

可知 β_v 普遍大于0.

即得出 $\beta_S>\beta_v>0$ 而结论.

$\delta T\delta S-\delta p\delta V>0.$

选 (p,V) 为独立变量.

$$\delta T=\left(\frac{\partial T}{\partial p}\right)_V\delta p+\left(\frac{\partial T}{\partial V}\right)_p\delta V$$

$$\delta S=\left(\frac{\partial S}{\partial p}\right)_V\delta p+\left(\frac{\partial S}{\partial V}\right)_p\delta V$$

$$\Rightarrow \left(\frac{\partial T}{\partial p}\right)_V\left(\frac{\partial S}{\partial p}\right)_V(\delta p)^2 +\left[\left(\frac{\partial T}{\partial V}\right)_p\left(\frac{\partial S}{\partial p}\right)_V+\left(\frac{\partial T}{\partial p}\right)_V\left(\frac{\partial S}{\partial V}\right)_p-1\right]\delta p\delta V + \left(\frac{\partial T}{\partial V}\right)_p\left(\frac{\partial S}{\partial V}\right)_p(\delta V)^2>0.$$

$$\Rightarrow\begin{cases}\left(\frac{\partial T}{\partial p}\right)_V\left(\frac{\partial S}{\partial p}\right)_V>0\\ \left(\frac{\partial T}{\partial V}\right)_p\left(\frac{\partial S}{\partial V}\right)_p>0\\ \left[\left(\frac{\partial T}{\partial V}\right)_p\left(\frac{\partial S}{\partial p}\right)_V+\left(\frac{\partial T}{\partial p}\right)_V\left(\frac{\partial S}{\partial V}\right)_p-1\right]^2 < \left(\frac{\partial T}{\partial p}\right)_V\left(\frac{\partial S}{\partial p}\right)_V\left(\frac{\partial T}{\partial V}\right)_p\left(\frac{\partial S}{\partial V}\right)_p\end{cases}$$

① $\left(\frac{\partial T}{\partial p}\right)_V\left(\frac{\partial S}{\partial p}\right)_V>0.$ $\quad\left(\frac{\partial S}{\partial p}\right)_V=-\left(\frac{\partial V}{\partial T}\right)_S.$

$$\frac{\partial(T,V)}{\partial(p,V)}\cdot\frac{\partial(S,V)}{\partial(p,V)}>0.$$

$\beta_V=\frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V.\quad \alpha_S=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_S.$

由

$$\frac{\left(\frac{\partial V}{\partial T}\right)_S}{\left(\frac{\partial p}{\partial T}\right)_V}<0.$$

$$\frac{V\alpha_S}{p\beta_V}<0.$$

$\boxed{\text{一般 } \beta_V>0.\quad \alpha_S<0}$

同理 ② $\left(\frac{\partial S}{\partial V}\right)_p=\left(\frac{\partial p}{\partial T}\right)_S.$

$$\frac{\left(\frac{\partial p}{\partial T}\right)_S}{\left(\frac{\partial V}{\partial T}\right)_p}>0.$$

$\beta_S=\frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_S.\quad \alpha_p=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p.$

$$\Rightarrow\frac{p\beta_S}{V\alpha_p}>0.$$

$\boxed{\text{一般 } \beta_S>0.\quad \alpha_p>0.}$

还有:

$$\left[\left(\frac{\partial T}{\partial V}\right)_p\left(\frac{\partial S}{\partial p}\right)_V+\left(\frac{\partial T}{\partial p}\right)_V\left(\frac{\partial S}{\partial V}\right)_p-1\right]^2<\left(\frac{\partial T}{\partial p}\right)_V\left(\frac{\partial S}{\partial p}\right)_V\left(\frac{\partial T}{\partial V}\right)_p\left(\frac{\partial S}{\partial V}\right)_p.$$

又由 $\frac{\partial(T,S)}{\partial(p,V)}=\left(\frac{\partial T}{\partial p}\right)_V\left(\frac{\partial S}{\partial V}\right)_p-\left(\frac{\partial T}{\partial V}\right)_p\left(\frac{\partial S}{\partial p}\right)_V=1.$

$$\Rightarrow\boxed{4\left[\left(\frac{\partial T}{\partial V}\right)_p\left(\frac{\partial S}{\partial p}\right)_V\right]^2<\left(\frac{\partial T}{\partial p}\right)_V\left(\frac{\partial S}{\partial p}\right)_V\left(\frac{\partial T}{\partial V}\right)_p\left(\frac{\partial S}{\partial V}\right)_p}$$

选 (T,S) 为独立变量.

$$\delta p=\left(\frac{\partial p}{\partial T}\right)_S\delta T+\left(\frac{\partial p}{\partial S}\right)_T\delta S.$$

$$\delta V=\left(\frac{\partial V}{\partial T}\right)_S\delta T+\left(\frac{\partial V}{\partial S}\right)_T\delta S.$$

$$\delta p\delta V-\delta T\delta S<0.$$

$$\left(\frac{\partial p}{\partial T}\right)_S\left(\frac{\partial V}{\partial T}\right)_S(\delta T)^2+\left[\left(\frac{\partial p}{\partial S}\right)_T\left(\frac{\partial V}{\partial T}\right)_S+\left(\frac{\partial p}{\partial T}\right)_S\left(\frac{\partial V}{\partial S}\right)_T-1\right]\delta T\delta S + \left(\frac{\partial p}{\partial S}\right)_T\left(\frac{\partial V}{\partial S}\right)_T(\delta S)^2<0.$$

$$\Rightarrow\begin{cases}\left(\frac{\partial p}{\partial T}\right)_S\left(\frac{\partial V}{\partial T}\right)_S<0\\ \left(\frac{\partial p}{\partial S}\right)_T\left(\frac{\partial V}{\partial S}\right)_T<0\\ \left[\left(\frac{\partial p}{\partial S}\right)_T\left(\frac{\partial V}{\partial T}\right)_S+\left(\frac{\partial p}{\partial T}\right)_S\left(\frac{\partial V}{\partial S}\right)_T-1\right]^2<\left(\frac{\partial p}{\partial T}\right)_S\left(\frac{\partial V}{\partial T}\right)_S\left(\frac{\partial p}{\partial S}\right)_T\left(\frac{\partial V}{\partial S}\right)_T\end{cases}$$

$\beta_S=\frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_S.$

$\alpha_S=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p.$ $\quad\boxed{\beta_S>0\Rightarrow\alpha_S<0.}$

$\left(\frac{\partial S}{\partial p}\right)_T=-\left(\frac{\partial V}{\partial T}\right)_p\qquad\left(\frac{\partial S}{\partial V}\right)_T=\left(\frac{\partial p}{\partial T}\right)_V$

$$\left(\frac{\partial S}{\partial p}\right)_T\left(\frac{\partial S}{\partial V}\right)_T<0.$$

$$\left(\frac{\partial V}{\partial T}\right)_p\left(\frac{\partial p}{\partial T}\right)_V>0.$$

$\alpha_p=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p\qquad\beta_V=\frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V.$

$\boxed{\beta_V>0\Rightarrow\alpha_p>0}$

$$4\ \frac{\partial(p,V)}{\partial(T,S)}=\left(\frac{\partial p}{\partial T}\right)_S\left(\frac{\partial V}{\partial S}\right)_T-\left(\frac{\partial p}{\partial S}\right)_T\left(\frac{\partial V}{\partial T}\right)_S=1.$$

$$\boxed{4\left[\left(\frac{\partial p}{\partial S}\right)_T\left(\frac{\partial V}{\partial T}\right)_S\right]^2<\left(\frac{\partial p}{\partial T}\right)_S\left(\frac{\partial V}{\partial T}\right)_S\left(\frac{\partial p}{\partial S}\right)_T\left(\frac{\partial V}{\partial T}\right)_T.}$$

LG公式单

《实验物理中的统计方法》

概率：频率；信心

- 贝叶斯：

$$p'(\varepsilon|\theta) = \frac{P(\theta|\varepsilon)\,P(\varepsilon)}{\int P(\theta|\varepsilon)\,P(\varepsilon)\,d\varepsilon}$$

θ. 事件.

$$P(A_j|B) = \frac{P(B|A_j)\,P(A_j)}{\sum_i P(B|A_i)P(A_i)}$$

- pdf：概率密度函数

cdf：累积分布函数

众数 ~ 最可几值

- 联合概率密度. $f(x_i)$

某一 x_i 的边缘概率密度.

$$f_x(x) = \int f(x,y)\,dy$$

$$g(x|y) = \frac{h(y|x)\,f(x)}{f_y(y)} = \frac{h(y|x)\,f(x)}{\int h(y|x)\,f(x)\,dx}$$

分布：$E[x] = \int x\,f(x)\,dx$

$$V[x] = \int (x - E[x])^2 f(x)\,dx$$

- 二项分布：

$$f(n;N,p) = \frac{N!}{n!(N-n)!}\,p^n(1-p)^{N-n}$$

$E[n] = Np \qquad V[n] = Np(1-p)$

- 均匀分布：$f(x;\alpha,\beta) = \frac{1}{\beta-\alpha}$

$E[x] = \frac{\alpha+\beta}{2} \qquad V[x] = \frac{1}{12}(\beta-\alpha)^2$

- 多项分布：

$$f(n_i;N,p_i) = \frac{N!}{n_1!\cdots n_m!}\,p_1^{n_1}\cdots p_m^{n_m}$$

$E[n_i] = Np_i$

$Cov[n_i,n_j] = Np_i(\delta_{ij} - p_j)$

- 泊松分布：

$$f(n;\nu) = \frac{\nu^n}{n!}e^{-\nu}$$

$E[n] = \nu \qquad V[n] = \nu$

- 指数分布：

$$f(x;\xi) = \frac{1}{\xi}e^{-x/\xi}\,dx$$

$E[x] = \xi \qquad V[x] = \xi^2$

- 卡方分布：

$$\chi^2(n) = \frac{z^{n/2-1}e^{-z/2}}{2^{n/2}\,\Gamma(n/2)} \qquad \Gamma(z) = \int_0^{+\infty} e^{-t}\,t^{z-1}\,dt$$

$E[z] = n \qquad V[z] = 2n$

- 高斯分布：

$$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$E[x] = \mu \qquad V[x] = \sigma^2$

- 伽马分布.

$$f(x;\alpha,\beta) = \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\,\Gamma(\alpha)} = \frac{\left(\frac{x}{\beta}\right)^\alpha e^{-x/\beta}\frac{1}{x}}{\Gamma(\alpha)}$$

$E[x] = \alpha\beta \qquad V[x] = \alpha\beta^2$

- 贝塔分布：

$$f(x;\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$$

$E[x] = \frac{\alpha}{\alpha+\beta} \qquad V[x] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

· 探测粒子:

$$\varepsilon = \frac{N'}{N},$$

$$\Delta N' = \sqrt{N\varepsilon(1-\varepsilon)}$$

$$\Delta\varepsilon = \frac{\Delta N'}{N}$$

不确定度传递.

$$U_{ij} = \frac{\partial y_i}{\partial x_m} V_{mn} \frac{\partial y_j}{\partial x_n}$$

变异系数: $C_v = \frac{\nu_2^{1/2}}{E[x]}$

偏度系数: $\beta_s = \frac{\nu_3}{\nu_2^{3/2}}$

- >0 正偏. 右偏
- <0 负偏. 左偏

峰度系数: $\beta_k = \frac{\nu_4}{\nu_2^2} - 3$

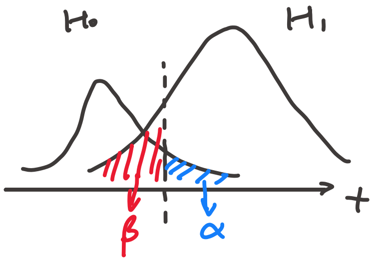
- >0 更尖
- <0 更宽

第一类错误, 弃真错误: α
(显著性水平. 检验的大小)

第二类错误、取伪错误: β (没有)

$1-\alpha$: 效率.

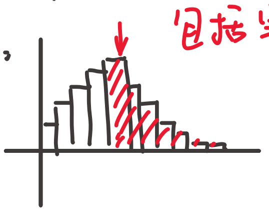
$1-\beta$: 功效



纯度: $P = \frac{\text{主信号占比}\cdot P(H_{主}|t\in T)}{\text{主}P(H_{主}|t\in T) + \text{其他}P(H_{其他}|t\in T)}$

(需指明区域T)

P值:



包括实验情况

(概率质量函数)

估计量: ① 相合性. ② 偏倚性. ③ 有效性.

①: $\lim_{n\to\infty}\hat\theta = \theta$ $\quad \lim_{n\to\infty} P(|\hat\theta-\theta|<\varepsilon)=0$, $\forall \varepsilon>0$.

②: $b = E[\hat\theta] - \theta = 0$

③: 对 $\forall \hat\theta'$, $\lim_{n\to\infty} \frac{V[\hat\theta_n]}{V[\hat\theta'_n]} \le 1$

最大似然法: 样本: $\vec{x} = (x_1, \cdots, x_n)$

估计量: $\vec\theta = (\theta_1, \cdots, \theta_m)$

$$L(\vec\theta) = f(\vec x;\vec\theta)$$

$$L(\vec\theta) = \prod_{i=1}^n f(x_i,\vec\theta) \quad \text{对独立情况.}$$

$$\ln L(\vec\theta) = \sum_{i=1}^n \ln f(x_i,\vec\theta)$$

def: $\hat\theta$: $\frac{\partial L(\vec\theta)}{\partial\theta_i} = 0$ → 极值中取最大值.

信息不等式: (RCF 不等式)

$$V[\hat\theta] \ge \left(1+\frac{\partial b}{\partial\theta}\right)^2 \Big/ E\left[-\frac{\partial^2 \ln L}{\partial\theta^2}\right]$$

b很小 ⇒ $V[\hat\theta] \approx -\frac{1}{E\left[\frac{\partial^2 \ln L}{\partial\theta^2}\right]}$ ⇒ 用于对 $V[\hat\theta]$ 的估计.

费舍尔信息矩阵:

$$V_{ij} = cov[\hat\theta_i, \hat\theta_j]$$

$$(V^{-1})_{ij} = -\frac{\partial^2 \ln L}{\partial\theta_i \partial\theta_j}\Big|_{\vec\theta=\hat{\vec\theta}} \quad (\text{近似}).$$

图解法:

$$\ln L = \ln L(\hat\theta) + \left[\frac{\partial \ln L}{\partial\theta}\right]_{\theta=\hat\theta}(\theta-\hat\theta) + \frac{1}{2!}\left[\frac{\partial^2 \ln L}{\partial\theta^2}\right]_{\theta=\hat\theta}(\theta-\hat\theta)^2$$

$$\left[\frac{\partial^2 \ln L}{\partial\theta^2}\right]_{\theta=\hat\theta} = \frac{1}{\hat\sigma^2_{\hat\theta}}$$

$$\Rightarrow \log L(\theta) \approx \log L_{max} - \frac{(\theta-\hat\theta)^2}{2\hat\sigma^2_{\hat\theta}}$$

$$\log L(\hat\theta \pm \hat\sigma_{\hat\theta}) \approx \log L_{max} - \frac{1}{2}$$

最小二乘法: (对高斯变量).

估计量 $\vec\theta$, 测量值 $\vec x$ (无误差或误差很小).

理论: $\lambda_i = \lambda(x_i, \vec\theta)$, ↔ 实际测量 y_i (有误差 σ_i)

$$\chi^2(\vec\theta) = \sum_{i=1}^N \frac{(y_i - \lambda(x_i;\vec\theta))^2}{\sigma_i^2}$$

一般情况.

$$\chi^2(\vec\theta) = \sum_{ij} [y_i - \lambda(x_i;\theta)](V^{-1})_{ij}[y_j - \lambda(x_j,\theta)]$$

线性: $\lambda \to \lambda(x;\vec\theta) = \sum_{j=1}^m a_j(x)\theta_j$

非线性: $\lambda \to \lambda(x;\vec\theta) \ne \sum_{j=1}^m a_j(x)\theta_j$

线性最小二乘.

令 $A_{ij} = a_j(x_i)$

$$\chi^2(\vec{\theta}) = (\vec{y} - \vec{\lambda})^T V^{-1} (\vec{y} - \vec{\lambda})$$

$$= (\vec{y} - A\vec{\theta})^T V^{-1} (\vec{y} - A\vec{\theta})$$

$$\nabla \chi^2 = -2 A^T V^{-1} (\vec{y} - A\vec{\theta}) = 0$$

$$\hat{\vec{\theta}} = (A^T V^{-1} A)^{-1} A^T V^{-1} \vec{y} = B\vec{y}$$

记 $U =$

一个有趣的积分恒等式.

$$\int_0^{+\infty} \frac{\eta^{z-\frac{1}{2}}}{(1+\eta)^{2z+\frac{1}{2}}}\,d\eta = \int_0^{+\infty} \frac{\eta^{z-1}}{(1+\eta)^{2z+\frac{1}{2}}}\,d\eta \quad \#$$

其中 z 可取 $\mathrm{Re}\,z>0$ 的所有复数.

有关证明如下:

定义: $\Gamma(z)=\int_0^{+\infty} e^{-t}t^{z-1}\,dt$; 构造 $\Gamma(z)\Gamma(z+\frac{1}{2})$.

则 $\Gamma(z)\Gamma(z+\frac{1}{2})=\int_0^{+\infty} t^{z-1}e^{-t}\,dt \int_0^{+\infty} s^{z-\frac{1}{2}}e^{-s}\,ds$.

$$=\iint_0^{+\infty} \left(\frac{t}{s}\right)^{z-1} e^{-(t+s)}\, s^{2z-\frac{3}{2}}\,dt\,ds \qquad ①$$

$$=\iint_0^{+\infty} \left(\frac{s}{t}\right)^{z-1} e^{-(s+t)}\, t^{2z-\frac{3}{2}}\,ds\,dt. \qquad ②$$

令 $\xi=t+s$, $\eta=t/s$. 则:

$\because t\in(0,+\infty)$ $s\in(0,+\infty)$.

$\xi\in(0,+\infty)$ $\eta\in(0,+\infty)$.

且有: $\begin{cases} t=\dfrac{\xi\eta}{1+\eta} \\ s=\dfrac{\xi}{1+\eta} \end{cases}$ ③

$$\left|\frac{\partial(t,s)}{\partial(\xi,\eta)}\right| = \left|\frac{\partial t}{\partial\xi}\frac{\partial s}{\partial\eta}-\frac{\partial t}{\partial\eta}\frac{\partial s}{\partial\xi}\right|$$

$$=\frac{\xi}{(1+\eta)^2} \qquad ④.$$

代入① ⇒

$$\Gamma(z)\Gamma(z+\tfrac{1}{2})=\iint_0^{+\infty}\eta^{z-1}e^{-\xi}\left(\frac{\xi}{1+\eta}\right)^{2z-\frac{3}{2}}\left|\frac{\partial(t,s)}{\partial(\xi,\eta)}\right|d\xi\,d\eta$$

$$=\int_0^{+\infty}d\xi\cdot\xi^{2z-\frac{1}{2}}e^{-\xi}\cdot\int_0^{+\infty}d\eta\,\frac{\eta^{z-1}}{(1+\eta)^{2z+\frac{1}{2}}}$$

$$=\Gamma(2z+\tfrac{1}{2})\int_0^{+\infty}\frac{\eta^{z-1}}{(1+\eta)^{2z+\frac{1}{2}}}\,d\eta \qquad ⑤.$$

代入② ⇒

$$\Gamma(z)\Gamma(z+\tfrac{1}{2})=\iint_0^{+\infty}\left(\frac{1}{\eta}\right)^{z-1}e^{-\xi}\left(\frac{\xi\eta}{1+\eta}\right)^{2z-\frac{3}{2}}\frac{\xi}{(1+\eta)^2}\,d\xi\,d\eta$$

$$=\int_0^{+\infty}d\xi\;\xi^{2z-\frac{1}{2}}e^{-\xi}\,d\xi\cdot\int_0^{+\infty}\frac{\eta^{z-\frac{1}{2}}}{(1+\eta)^{2z+\frac{1}{2}}}\,d\eta$$

$$=\Gamma(2z+\tfrac{1}{2})\int_0^{+\infty}\frac{\eta^{z-\frac{1}{2}}}{(1+\eta)^{2z+\frac{1}{2}}}\,d\eta \qquad ⑥.$$

比较⑤.⑥式. 可知 ⇒.

$$\Gamma(z)\Gamma(z+\tfrac{1}{2})=\Gamma(2z+\tfrac{1}{2})\int_0^{+\infty}\frac{\eta^{z-1}}{(1+\eta)^{2z+\frac{1}{2}}}\,d\eta$$

$$=\Gamma(2z+\tfrac{1}{2})\int_0^{+\infty}\frac{\eta^{z-\frac{1}{2}}}{(1+\eta)^{2z+\frac{1}{2}}}\,d\eta$$

由此推知 ⇒

$$\int_0^{+\infty}\frac{\eta^{z-1}}{(1+\eta)^{2z+\frac{1}{2}}}\,d\eta=\int_0^{+\infty}\frac{\eta^{z-\frac{1}{2}}}{(1+\eta)^{2z+\frac{1}{2}}}\,d\eta$$

结论得证.

其实, 这个恒等式还有一个更简洁的证法,

我将他写在背面...

$$A = \int_0^{+\infty} \frac{\eta^{z-\frac{1}{2}}}{(1+\eta)^{2z+\frac{1}{2}}} d\eta$$

令 $\xi = \frac{1}{\eta}$. $d\eta = -\frac{1}{\xi^2} d\xi$, $\xi \in (0, +\infty)$.

$$A = \int_{+\infty}^{0} \frac{\left(\frac{1}{\xi}\right)^{z-\frac{1}{2}}}{\left(1+\frac{1}{\xi}\right)^{2z+\frac{1}{2}}} \left(-\frac{1}{\xi^2}\right) d\xi.$$

$$= \int_0^{+\infty} \frac{\xi^{z-1}}{(\xi+1)^{2z+\frac{1}{2}}} d\xi.$$

得证.

沿空间任意方向运动导致的洛伦兹变换矩阵

· 所求：S系 $\xrightarrow{\vec{v}\text{-boost}}$ S'系.

S系坐标 $x^{\mu} = (t, x, y, z)$.

S'系坐标 $x^{\mu'} = (t', x', y', z')$. (0,1)

$x^{\mu'} = \Lambda^{\mu'}{}_{\nu} x^{\nu}$，$\Lambda^{\mu'}{}_{\nu}$形式？

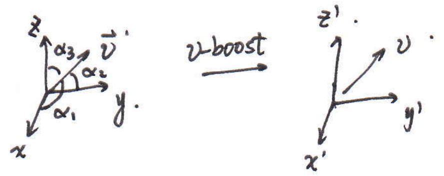
· 已知：$\vec{v} = (0, v_1, v_2, v_3)$. (0,2)

$v_i = \cos\alpha_i \cdot v$，且规定 $\alpha_i \in [0, \frac{\pi}{2}]$ (0,3)

x-boost：$\Lambda_x{}^{\mu'}{}_{\nu} = \begin{bmatrix} \gamma & -\gamma\beta & & \\ -\gamma\beta & \gamma & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$ (0,4)

· 最大困难：

(i)



S'坐标在变换后"摆放"方式.

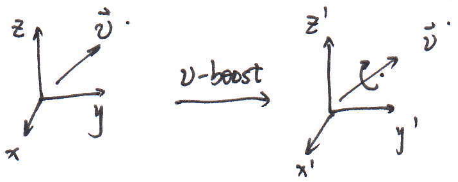
假定：$\vec{v}$的方向向量 $(\cos\alpha_1, \cos\alpha_2, \cos\alpha_3) = \hat{n}$. (0,5)

在变换前后 $(\hat{n})_S = (\hat{n}')_{S'}$.

(ii) 将 v-boost 通过 $\vec{R}(\theta)$. 两次变换后.

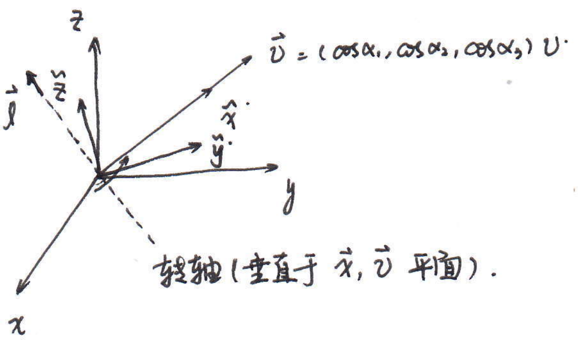
坐标系是否有绕 $\vec{v}$ 的转动.

亦即要保证：



· 正文.

一、先算 α 转动矩阵 $\vec{R}(\alpha_1)$：

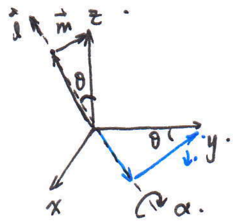


$$\vec{l} = \hat{x} \times \hat{n} = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \end{vmatrix} = (0, -\cos\alpha_3, \cos\alpha_2). \quad (1,1)$$

$$\hat{l} = \frac{\vec{l}}{|\vec{l}|} = \frac{(0, -\cos\alpha_3, \cos\alpha_2)}{\sqrt{1-\cos^2\alpha_1}} \quad (1.2)$$

$\check{S}$ 较 S. 绕 $\vec{l}$ 转 α_1 大小.

$\{\hat{x}, \hat{y}, \hat{z}\} \to \{\hat{\check{x}}, \hat{\check{y}}, \hat{\check{z}}\}$.



$$\hat{z} = \cos\theta\, \hat{l} + \sin\theta\, \hat{m}$$
$$\hat{\check{z}} = \cos\theta\, \hat{l} + \sin\theta\, \hat{\check{m}} \quad (1.3)$$

$$\hat{\check{z}} = \cos^2\theta\, \hat{z} - \cos\theta\sin\theta\, \hat{y} + \sin\theta\sin\alpha\,(-\hat{x}) + \sin\theta\cos\alpha\,(\sin\theta\,\hat{z} + \cos\theta\,\hat{y}).$$
$$= -\sin\theta\sin\alpha\,\hat{x} + \sin\theta\cos\theta(\cos\alpha - 1)\,\hat{y} + (\cos^2\theta + \sin^2\theta\cos\alpha)\,\hat{z}. \quad (1.4)$$

$$\hat{\check{y}} = \sin^2\theta\, \hat{y} - \sin\theta\cos\theta\, \hat{z} + \cos\theta\sin\alpha\,(-\hat{x}) + \cos\theta\cos\alpha \cdot (\sin\theta\,\hat{z} + \cos\theta\,\hat{y}).$$
$$= -\cos\theta\sin\alpha\,\hat{x} + (\sin^2\theta + \cos^2\theta\cos\alpha)\,\hat{y} + \sin\theta\cos\theta(\cos\alpha - 1)\,\hat{z}. \quad (1.5)$$

$$\hat{\check{x}} = \cos\alpha_1\,\hat{x} + \cos\alpha_2\,\hat{y} + \cos\alpha_3\,\hat{z}. \quad (1.6)$$

于是，
$$\vec{R}(\alpha_1) = \begin{bmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \\ -\cos\theta\sin\alpha & \sin^2\theta + \cos^2\theta\cos\alpha & \sin\theta\cos\theta(\cos\alpha - 1) \\ -\sin\theta\sin\alpha & \sin\theta\cos\theta(\cos\alpha - 1) & \cos^2\theta + \sin^2\theta\cos\alpha \end{bmatrix} \quad (1.7)$$

下算 θ.

$$\begin{cases} \cos\theta = \dfrac{\hat{l} \cdot \hat{z}}{|\hat{l}| \cdot |\hat{z}|} = \dfrac{\cos\alpha_2}{\sqrt{1-\cos^2\alpha_1}} \\[2ex] \sin\theta = \sqrt{1-\cos^2\theta} = \dfrac{\cos\alpha_3}{\sqrt{1-\cos^2\alpha_1}} \end{cases} \quad (1.8)$$

且 $\alpha = \alpha_1$

(1.8) 代入 (1.7).

$$\vec{R}(\alpha_1) = \begin{bmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \\ -\dfrac{\sin\alpha_1\cos\alpha_2}{\sqrt{1-\cos^2\alpha_1}} & \dfrac{\cos\alpha_1\cos^2\alpha_2 + \cos^2\alpha_3}{1-\cos^2\alpha_1} & \dfrac{\cos\alpha_2\cos\alpha_3(\cos\alpha_1 - 1)}{1-\cos^2\alpha_1} \\ -\dfrac{\sin\alpha_1\cos\alpha_3}{\sqrt{1-\cos^2\alpha_1}} & \dfrac{\cos\alpha_2\cos\alpha_3(\cos\alpha_1 - 1)}{1-\cos^2\alpha_1} & \dfrac{\cos^2\alpha_2 + \cos^2\alpha_3\cos\alpha_1}{1-\cos^2\alpha_1} \end{bmatrix} \quad (1.9)$$

利用 $|\sin\alpha_1| = \sqrt{1-\cos^2\alpha_1}$ 化简 (1.9).

$$\vec{R}(\alpha_1) = \begin{bmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \\ -\frac{\sin\alpha_1}{|\sin\alpha_1|}\cos\alpha_2 & \frac{\cos\alpha_1\cos^2\alpha_2+\cos^2\alpha_3}{\sin^2\alpha_1} & \frac{\cos\alpha_2\cos\alpha_3(\cos\alpha_1-1)}{\sin^2\alpha_1} \\ -\frac{\sin\alpha_1}{|\sin\alpha_1|}\cos\alpha_3 & \frac{\cos\alpha_2\cos\alpha_3(\cos\alpha_1-1)}{\sin^2\alpha_1} & \frac{\cos^2\alpha_2+\cos^2\alpha_3\cos\alpha_1}{\sin^2\alpha_1} \end{bmatrix} \quad (1.10).$$

易于验证 (1.10) 式满足

$$\vec{R}(\alpha_1)^T = \vec{R}(\alpha_1)^{-1} \quad (\vec{R}^{-1}(\alpha_1) = \vec{R}(-\alpha_1)).$$

下面将 α_i 定义域限于 $[0, \frac{\pi}{2}]$ ⇒.

$$\vec{R}(\alpha_1) = \begin{bmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \\ -\cos\alpha_2 & \frac{\cos\alpha_1\cos^2\alpha_2+\cos^2\alpha_3}{\sin^2\alpha_1} & \frac{\cos\alpha_2\cos\alpha_3(\cos\alpha_1-1)}{\sin^2\alpha_1} \\ -\cos\alpha_3 & \frac{\cos\alpha_2\cos\alpha_3(\cos\alpha_1-1)}{\sin^2\alpha_1} & \frac{\cos^2\alpha_2+\cos^2\alpha_3\cos\alpha_1}{\sin^2\alpha_1} \end{bmatrix} \quad (1.11).$$

二、计算. $S \to S'$ 的变换 $\Lambda^{\mu'}{}_{\eta}$

$$x^{\mu'} = [\vec{R}(\alpha_1)^{-1}]^{\mu'}{}_{\nu'} \vec{\Lambda}_x{}^{\nu'}{}_{\sigma} [\vec{R}(\alpha_1)]^{\sigma}{}_{\eta} X^{\eta} \quad (2.1)$$

$$\Lambda^{\mu'}{}_{\eta} = \vec{R}(\alpha_1)_{\nu'}{}^{\mu'} \vec{\Lambda}_x{}^{\nu'}{}_{\sigma} \vec{R}(\alpha_1)^{\sigma}{}_{\eta}. \quad (2.2).$$

经计算:

$$\vec{\Lambda}_x{}^{\nu'}{}_{\sigma} \vec{R}(\alpha_1)^{\sigma}{}_{\eta} = \quad (2.3)$$

$$= \begin{bmatrix} \gamma & -\gamma\beta\cos\alpha_1 & -\gamma\beta\cos\alpha_2 & -\gamma\beta\cos\alpha_3 \\ -\gamma\beta & \gamma\cos\alpha_1 & \gamma\cos\alpha_2 & \gamma\cos\alpha_3. \\ 0 & -\cos\alpha_2. & \frac{\cos\alpha_1\cos^2\alpha_2+\cos^2\alpha_3}{\sin^2\alpha_1} & \frac{\cos\alpha_2\cos\alpha_3(\cos\alpha_1-1)}{\sin^2\alpha_1} \\ 0 & -\cos\alpha_3 & \frac{\cos\alpha_2\cos\alpha_3(\cos\alpha_1-1)}{\sin^2\alpha_1} & \frac{\cos\alpha_1\cos^2\alpha_2+\cos^2\alpha_3}{\sin^2\alpha_1} \end{bmatrix}$$

$$[\vec{R}(\alpha_1)^{-1}]^{\mu'}{}_{\nu'} \vec{\Lambda}_x{}^{\nu'}{}_{\sigma} \vec{R}(\alpha_1)^{\sigma}{}_{\eta} = \Lambda^{\mu'}{}_{\eta} =$$

$$= \begin{bmatrix} \gamma & -\gamma\beta\cos\alpha_1. & -\gamma\beta\cos\alpha_2. & -\gamma\beta\cos\alpha_3. \\ -\gamma\beta\cos\alpha_1 & \gamma\cos^2\alpha_1+\cos^2\alpha_2+\cos^2\alpha_3; & \gamma\cos\alpha_1\cos\alpha_2 - \frac{\cos\alpha_1\cos^3\alpha_2+\cos^3\alpha_3\cos\alpha_2}{\sin^2\alpha_1} - \frac{\cos\alpha_2\cos^3\alpha_3(\cos\alpha_1-1)}{\sin^2\alpha_1}. & \Lambda^{1'}{}_3 \\ -\gamma\beta\cos\alpha_2 & \boxed{} & \gamma\cos^2\alpha_2 + \frac{(\cos\alpha_1\cos^2\alpha_2+\cos^2\alpha_3)^2}{\sin^4\alpha_1} + \frac{\cos^2\alpha_2\cos^2\alpha_3(\cos\alpha_1-1)^2}{\sin^4\alpha_1}. & \Lambda^{2'}{}_3 \\ -\gamma\beta\cos\alpha_3. & \Lambda^{3'}{}_1 & \Lambda^{2'}{}_3. & \Lambda^{3'}{}_3. \end{bmatrix} \quad (2.4)$$

$$\Lambda^{1'}{}_3 = \Lambda^{3'}{}_1 = \gamma\cos\alpha_1\cos\alpha_3 - \frac{\cos^2\alpha_2\cos\alpha_3(\cos\alpha_1-1)+\cos^2\alpha_2\cos\alpha_3+\cos^3\alpha_3\cos\alpha_1}{\sin^2\alpha_1}$$

$$= \gamma\cos\alpha_1\cos\alpha_3 - \frac{(1-\cos^2\alpha_1-\cos^2\alpha_3)\cos\alpha_3(\cos\alpha_1-1)+(1-\cos^2\alpha_1-\cos^2\alpha_3)\cos\alpha_3+\cos^3\alpha_3\cos\alpha_1}{1-\cos^2\alpha_1}$$

$$= \gamma\cos\alpha_1\cos\alpha_3 - \frac{1}{1-\cos^2\alpha_1}\left[(1-\cos^2\alpha_1)[\cos\alpha_3(\cos\alpha_1-1)+\cos\alpha_3]\right].$$

$$= \gamma\cos\alpha_1\cos\alpha_3 - \frac{1}{\cancel{1-\cos^2\alpha_1}}\cdot\cancel{(1-\cos^2\alpha_1)}\cos\alpha_1\cos\alpha_3.$$

$$= (\gamma-1)\cos\alpha_1\cos\alpha_3. \quad (2.5).$$

※

$$\Lambda^{2'}{}_3 = \Lambda^{3'}{}_2 = \gamma\cos\alpha_2\cos\alpha_3 + \frac{(\cos\alpha_1\cos^2\alpha_2+\cos^2\alpha_3)(\cos\alpha_2\cos\alpha_3(\cos\alpha_1-1))}{(\cos^2\alpha_2+\cos^2\alpha_3)^2}.$$

$$+ \frac{(\cos^2\alpha_2+\cos\alpha_1\cos^2\alpha_3)(\cos\alpha_2\cos\alpha_3(\cos\alpha_1-1))}{(\cos^2\alpha_2+\cos^2\alpha_3)^2}.$$

$$= \gamma\cos\alpha_2\cos\alpha_3 + \frac{1}{(\cos^2\alpha_2+\cos^2\alpha_3)^2}\cdot\Big[\cos^2\alpha_1\cos^3\alpha_2\cos\alpha_3 - \cos\alpha_1\cancel{\cos^3\alpha_2\cos\alpha_3}$$

$$+ \cos\alpha_1\cancel{\cos\alpha_2\cos^3\alpha_3} - \cos\alpha_2\cos^3\alpha_3 + \cos\alpha_1\cancel{\cos^3\alpha_2\cos\alpha_3}.$$

$$- \cos^3\alpha_2\cos\alpha_3 + \cos^2\alpha_1\cos\alpha_2\cos^3\alpha_3 - \cos\alpha_1\cancel{\cos\alpha_2\cos^3\alpha_3}\Big].$$

$$= \gamma\cos\alpha_2\cos\alpha_3 + \frac{1}{(\cos^2\alpha_2+\cos^2\alpha_3)^2}\cdot\Big[(1-\cos^2\alpha_2-\cos^2\alpha_3)\cos\alpha_2\cos\alpha_3\cdot(\cos^2\alpha_2+\cos^2\alpha_3).$$

$$- \cos\alpha_2\cos\alpha_3[\cos^2\alpha_2+\cos^2\alpha_3].\Big].$$

$$= \gamma\cos\alpha_2\cos\alpha_3 + \frac{1}{(\cos^2\alpha_2+\cos^2\alpha_3)}\cdot\Big\{\cancel{\cos\alpha_2\cos\alpha_3} - (\cos^2\alpha_2+\cos^2\alpha_3)\cos\alpha_2\cos\alpha_3$$

$$- \cancel{\cos\alpha_2\cos\alpha_3}.\Big\}.$$

$$= \gamma\cos\alpha_2\cos\alpha_3 - \frac{1}{(\cos^2\alpha_2+\cos^2\alpha_3)^2}\cdot(\cos^2\alpha_2+\cos^2\alpha_3)\cos\alpha_2\cos\alpha_3$$

$$= (\gamma-1)\cos\alpha_2\cos\alpha_3. \quad (2.6)$$

$$\Lambda^{3'}{}_3 = \gamma\cos^2\alpha_3 + \frac{\cos^2\alpha_2\cos^2\alpha_3(\cos\alpha_1-1)^2}{\sin^4\alpha_1}$$

交叉项可消.

※

$$+ \frac{(\cos^2\alpha_2+\cos^2\alpha_3\cos\alpha_1)^2\,\cancel{(\cos^2\alpha_2\cos\alpha_1+\cos^2\alpha_3)}}{\sin^4\alpha_1}$$

$$= \gamma\cos^2\alpha_3 + \frac{\cos^2\alpha_2\cos^2\alpha_3\cos^2\alpha_1 + \cos^2\alpha_2\cos^2\alpha_3.}{\sin^4\alpha_1.}$$

$$+ \frac{\cos^4\alpha_2+\cos^4\alpha_3\cos^2\alpha_1.}{\sin^4\alpha_1.}$$

$$= \gamma\cos^2\alpha_3 + \frac{(1-\cos^2\alpha_3)(\cos^2\alpha_2+\cos^2\alpha_3)}{\sin^4\alpha_1}.$$

$$= (\gamma-1)\cos^2\alpha_3 + 1 \quad (2.7)$$

$$\Lambda^{1'}{}_{2} = \Lambda^{2'}{}_{1} =$$

$$= \gamma\cos\alpha_1\cos\alpha_2 - \frac{\cos\alpha_1\cos^3\alpha_2 + \cos\alpha_2\cos^2\alpha_3}{\sin^2\alpha_1} - \frac{\cos\alpha_2\cos^2\alpha_3(\cos\alpha_1-1)}{\sin^2\alpha_1}$$

$$= \gamma\cos\alpha_1\cos\alpha_2 - \frac{1}{1-\cos^2\alpha_1}\cdot\{\cos\alpha_1\cos^3\alpha_2 + \cos\alpha_2(1-\cos^2\alpha_1-\cos^2\alpha_2)$$

$$+ \cos\alpha_2(1-\cos^2\alpha_1-\cos^2\alpha_2)(\cos\alpha_1-1)\}.$$

$$= \gamma\cos\alpha_1\cos\alpha_2 - \frac{1}{1-\cos^2\alpha_1}\{(\cos\alpha_1-\cancel{1})\cos^3\alpha_2 + \cos\alpha_2(1-\cos^2\alpha_1).$$

$$+ \cos\alpha_2(\cos\alpha_1-1)(1-\cos^2\alpha_1) - \cos^3\alpha_2(\cancel{\cos\alpha_1}-1)\}.$$

$$= \gamma\cos\alpha_1\cos\alpha_2 - \frac{1}{1-\cancel{\cos^2\alpha_1}}\ \cos\alpha_1\cos\alpha_2\ (1-\cancel{\cos^2\alpha_1}).$$

$$= (\gamma-1)\cos\alpha_1\cos\alpha_2. \qquad (2.8)$$

$$\Lambda^{1'}{}_{1} = (\gamma-1)\cos^2\alpha_1 + (\cos^2\alpha_1+\cos^2\alpha_2+\cos^2\alpha_3).$$

$$= (\gamma-1)\cos^2\alpha_1 + 1. \qquad (2.9).$$

$$\Lambda^{2'}{}_{2} = \gamma\cos^2\alpha_2 + \frac{1}{\sin^4\alpha_1}\{(\cos\alpha_1\cos^2\alpha_2+\cos^2\alpha_3)^2$$

$$+ \cos^2\alpha_2\cos^2\alpha_3(\cos\alpha_1-1)^2\}.$$

$$= \gamma\cos^2\alpha_2 + \frac{1}{(1-\cos^2\alpha_1)^2}\{\cos^2\alpha_1\cos^4\alpha_2 + 2\cos\alpha_1\cos^2\alpha_2(1-\cos^2\alpha_1-\cos^2\alpha_2).$$

$$+ (1-\cos^2\alpha_1-\cos^2\alpha_2)^2 + \cos^2\alpha_2(1-\cos^2\alpha_1-\cos^2\alpha_2)(\cos\alpha_1-1)^2\}.$$

$$= \gamma\cos^2\alpha_2 + \frac{1}{(1-\cos^2\alpha_1)^2}\{\cos^2\alpha_1\cos^4\alpha_2 + 2\cos\alpha_1\cos^2\alpha_2(1-\cos^2\alpha_1).$$

$$-2\cos\alpha_1\cos^4\alpha_2 + (1-\cos^2\alpha_1)^2 - 2\cos^2\alpha_2(1-\cos^2\alpha_1) + \cos^4\alpha_2.$$

$$+ \cos^2\alpha_2(1-\cos^2\alpha_1)(\cos\alpha_1-1)^2 - \cos^4\alpha_2(\cos\alpha_1-1)^2\}.$$

$$= \gamma\cos^2\alpha_2 + \frac{1}{1-\cos^2\alpha_1}\{2\cos\alpha_1\cancel{\cos^2}\alpha_2 + (1-\cos^2\alpha_1) - 2\cos^2\alpha_2.$$

$$+ \cos^2\alpha_2(\cos^2\alpha_1+1-2\cancel{\cos}\alpha_1).\}$$

$$= \gamma\cos^2\alpha_2 + \frac{1}{1-\cos^2\alpha_1}\{(1-\cos^2\alpha_1) + \cos^2\alpha_2(\cos^2\alpha_1-1)\}$$

$$= (\gamma-1)\cos^2\alpha_2 + 1. \qquad (2.10)$$

综合 (2.4) ~ (2.10).

$$\Lambda^{\mu'}{}_{\eta} = \begin{bmatrix} \gamma & -\gamma\beta\cos\alpha_1 & -\gamma\beta\cos\alpha_2 & -\gamma\beta\cos\alpha_3 \\ -\gamma\beta\cos\alpha_1 & (\gamma-1)\cos^2\alpha_1+1 & (\gamma-1)\cos\alpha_1\cos\alpha_2 & (\gamma-1)\cos\alpha_1\cos\alpha_3 \\ -\gamma\beta\cos\alpha_2 & (\gamma-1)\cos\alpha_1\cos\alpha_2 & (\gamma-1)\cos^2\alpha_2+1 & (\gamma-1)\cos\alpha_2\cos\alpha_3 \\ -\gamma\beta\cos\alpha_3 & (\gamma-1)\cos\alpha_1\cos\alpha_3 & (\gamma-1)\cos\alpha_2\cos\alpha_3 & (\gamma-1)\cos^2\alpha_3+1 \end{bmatrix}. \qquad (2.11)$$

$$\boxed{\begin{aligned} &\Lambda^{0'}{}_{0} = \gamma \qquad \Lambda^{0'}{}_{i} = \Lambda^{i'}{}_{0} = -\gamma v^i/c \\ &\Lambda^{i'}{}_{j} = \Lambda^{j'}{}_{i} = (\gamma-1)\frac{v^i v^j}{|\vec{v}|^2} + \delta^{ij} \end{aligned}}$$

---- 总结.

三、总结.

(i) 观察 (2.11) 可见 $[\Lambda^{-1}(\vec{v})]^{\mu'}{}_{\eta} = [\Lambda(-\vec{v})]^{\mu'}{}_{\eta}$. (3.1)

由 $x^{\mu'} = \Lambda(\vec{v})^{\mu'}{}_{\eta}x^{\eta}$. (3.2)

$$\Lambda^{-1}x' = \Lambda^{-1}\Lambda X = X. \qquad (3.3)$$

$$X'^{T} = X^{T}\Lambda^{T} \qquad (3.4)$$

$$(X', X') = X'^{T}\eta X' = (\Lambda X)^{T}\eta(\Lambda X) \qquad (3.5).$$

$$= X^{T}(\Lambda^{T}\eta\Lambda)X = X^{T}\eta X = (X, X)$$

$$\Rightarrow \Lambda^{T}\eta\Lambda = \eta \Rightarrow \det(\Lambda^{T}\eta\Lambda) = \det(\eta) \qquad (3.6)$$

即 $\det\Lambda^{T}\det\eta\det\Lambda = \det\eta$ (3.7).

$$\Rightarrow [\det(\Lambda)]^2 = 1 \qquad (3.8)$$

(ii) 计算过程中. 求解 $\overleftrightarrow{R}(\alpha_1)$ 时.

由于使用的方法为先求出 $\check{x} = R\hat{x}$. (R 仅为一旋转阵)

$$\hat{x}^{T}\check{x} = \hat{x}^{T}x \qquad (3.9)$$

$$(R\hat{x})^{T}\check{x} = \hat{x}^{T}x.$$

$$\hat{x}^{T}(R^{T}\check{x}) = \hat{x}^{T}x. \Rightarrow \hat{x}^{T}(R^{T}\check{x}-x) = \hat{0}^{T}.$$

$\hat{x}^{T} = (1,1,1)$ 故 $R^{T}\check{x} - x = \hat{0}$.

$$\therefore R^{T}\check{x} = x \qquad (3.10)$$

此时需先证明 $R^{T} = R^{-1}$ (验证). (3.11)

再有 $\check{x} = (R^{T})^{-1}x = Rx$. (3.12)

得出 $R = \overleftrightarrow{R}(\alpha_1)$ 即为所求.

(iii) 易错点: $\overleftrightarrow{R}_{yy}$ 和 $\overleftrightarrow{R}_{zz}$ 并不完全相等.

(iv) 小技巧: 计算 Λ_{ij} 时. 可以先将 α_k 用 α_i, α_j 表示

再配凑因子与分母相消.

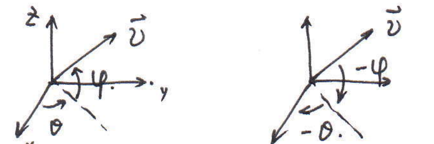
(v) 亿点说明: (对于困难 (ii)).

这里需要只能做一次转动. 即 $R^{-1}(\alpha)T(\eta)R(\alpha)$.

所有旋转对 $\check{x}$ 轴、$\hat{x}$ 轴、$\vec{v}$ 始终位于同一

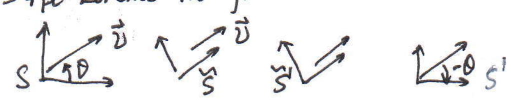
二维平面. 以保证不会生成 $\vec{v}$ 旋转

若是 $R^{-1}(\theta)R^{-1}(\varphi)T(\eta)R(\varphi)R(\theta)$ 则无法保证这一点.



详细记述类比 $(ct, x, y) \xrightarrow{\theta\text{-boost}} (ct', x', y')$

的二维 Lorentz Transform



LG 四大公式单.

• 关于 Legendre 方程 & Legendre 多项式.

1. $\nabla^2\varphi=0$，取球坐标时. ①.

* nabla 技巧：记 $d\vec r = h_1 dx_1\hat e_1 + h_2 dx_2 \hat e_2 + h_3 dx_3 \hat e_3$

• 梯度：$\nabla\varphi = \sum_{cyc}\frac{1}{h_1}\partial_1\varphi$.

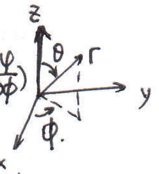
• $\nabla\times\vec A = \frac{1}{h_1h_2h_3}\begin{vmatrix}\hat e_1&\hat e_2&\hat e_3\\ \partial_1&\partial_2&\partial_3\\ h_1A_1&h_2A_2&h_3A_3\end{vmatrix}$

• 散度：$\nabla\cdot\vec A = \frac{1}{h_1h_2h_3}\sum_{cyc}\partial_1(h_2h_3A_1)$

⇒ • Laplace 算子：$\nabla^2\varphi = \frac{1}{h_1h_2h_3}\sum_{cyc}\partial_1\left(\frac{h_2h_3}{h_1}\partial_1\varphi\right)$

故 $h_r=1,\ h_\theta=r,\ h_\phi=r\sin\theta$.

$$0=\frac{\partial}{\partial r}\left(r^2\sin\theta\frac{\partial\varphi}{\partial r}\right)+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right)+\frac{\partial}{\partial\phi}\left(\frac{1}{\sin\theta}\frac{\partial\varphi}{\partial\phi}\right)$$



令 $\varphi(r,\theta,\phi)=R(r)\cdot\Theta(\theta)\cdot\Phi(\phi)$

$$\frac{2r\sin\theta R'}{R}+\frac{r^2\sin\theta R''}{R}+\frac{\cos\theta\,\Theta'}{\Theta}+\sin\theta\frac{\Theta''}{\Theta}+\frac{1}{\sin\theta}\frac{\Phi''}{\Phi}=0.$$

同除 $\sin\theta$：

$$\frac{2rR'}{R}+\frac{r^2R''}{R}+\frac{\cos\theta}{\sin\theta}\frac{\Theta'}{\Theta}+\frac{\Theta''}{\Theta}+\frac{1}{\sin^2\theta}\frac{\Phi''}{\Phi}=0. \quad ②$$

若等式恒成立：

$$\frac{2rR'}{R}+\frac{r^2R''}{R}=\lambda \quad (\lambda\text{定值}). \quad ③$$

$$\frac{\cos\theta}{\sin\theta}\frac{\Theta'}{\Theta}+\frac{\Theta''}{\Theta}+\frac{1}{\sin^2\theta}\frac{\Phi''}{\Phi}=-\lambda \quad ④.$$

③ ⇒ $r^2R''+2rR'-\lambda R=0$.

令 $R(r)=\sum_n a_n r^n$，有：（n 为整数）

$$\sum n(n-1)a_nr^n+\sum 2na_nr^n-\lambda\sum a_nr^n=0.$$

⇒ $[\lambda-n(n+1)]a_n=0$ ③#

∴ $\lambda=n(n+1)$ ⑤. $n\in\mathbb{Z}$. → λ 取值.

⑤代入④

$$\sin\theta\cos\theta\frac{\Theta'}{\Theta}+\sin^2\theta\frac{\Theta''}{\Theta}+\lambda\sin^2\theta+\frac{\Phi''}{\Phi}=0.$$

⇒ $\frac{\Phi''}{\Phi}=-\mu$ ⑥；$\sin\theta\cos\theta\frac{\Theta'}{\Theta}+\sin^2\theta\frac{\Theta''}{\Theta}+\lambda\sin^2\theta-\mu=0$ ⑦.

由⑦ 令 $x=\cos\theta$, $y(x)=\Theta(\theta)$.

⇒ $dx=-\sin\theta\,d\theta$.

$y'=\frac{dy}{dx}=-\frac{d\Theta}{\sin\theta\,d\theta}=-\frac{1}{\sin\theta}\Theta'$

$y''=\frac{d}{dx}\left(-\frac{1}{\sin\theta}\Theta'\right)=-\frac{1}{\sin\theta}\cdot\frac{d}{d\theta}\left(-\frac{1}{\sin\theta}\Theta'\right)$ $=-\frac{\cos\theta}{\sin^3\theta}\Theta'+\frac{1}{\sin^2\theta}\Theta''$

⇒ $2\sin^2\theta\cos\theta\, y'+\sin^4\theta\, y''+(\lambda\sin^2\theta-\mu)y=0$.

$(1-x^2)y''-2xy'+\left(\lambda-\frac{\mu}{1-x^2}\right)y=0$.

⇒ 特殊当 $\mu=0$ 时.（φ 在 φ 上有对称性）.

令 $y=\sum C_nx^n$

$$\sum(1-x^2)n(n-1)C_nx^{n-2}-2xC_n\cdot n\,x^{n-1}+\lambda C_nx^n=0.$$

⇒ $\sum -n(n-1)C_nx^n+(n+2)(n+1)C_{n+2}x^n-2nC_nx^n+\lambda C_nx^n=0$

⇒ $-n(n-1)C_n+(n+2)(n+1)C_{n+2}-2nC_n+\lambda C_n=0$.

⇒ $C_{n+2}=\frac{n(n+1)-\lambda}{(n+2)(n+1)}\cdot C_n$. ⑧

记 $\lambda=l(l+1)$. ⑨

则 $y=C_lP_l(x)$. ⑩ | *: C_l 非 C_n. C_l 为 $\lambda=l(l+1)$ 的系列解的集合前系数.

⇒ $\Theta(\theta)=C_lP_l(\cos\theta)$. ⑪

由 ③# ⇒. 令

当 $\lambda=l(l+1)$ a_n ✓

当 $\lambda\neq l(l+1)$ $a_n=0$.

⇒ $n=l$ 或 $n=-(l+1)$ a_n ✓

⇒ $R(r)=a_lr^l+b_lr^{-(l+1)}$ ⑫.

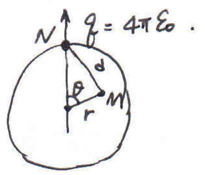
⑪⑫ ⇒

∴ $\varphi=\sum_l\left(a_lr^l+\frac{b_l}{r^{l+1}}\right)P_l(\cos\theta)$. ⑬##

＜需要φ与φ无关＞

∵ $P_l(\cos\theta)$ 为一正交归一完备基.

Legendre 多项式的母函数见背面.



N处有 $q=4\pi\varepsilon_0$. M电势 $v(r,\theta)$.

$$v(r,\theta)=\frac{1}{d}=\frac{1}{\sqrt{1-2r\cos\theta+r^2}}$$

令 $x=\cos\theta$.

$$\Rightarrow v(r,\theta)=\frac{1}{d}=\frac{1}{\sqrt{1-2rx+r^2}},\quad |x|\le 1 \qquad ⑭$$

单位球内.

$$\nabla^2 v(r,\theta)=0,\quad r<1 .$$

$$\Rightarrow \text{解得: } v(r,\theta)=\sum_{l=0}^{\infty}\left(A_l r^l+B_l r^{-(l+1)}\right)P_l(\cos\theta).$$

$r\to 0$, v 有限 . $\Rightarrow B_l=0$.

$$\Rightarrow v(r,\theta)=\sum_{l=0}^{\infty}A_l r^l P_l(\cos\theta),\quad r<1. \qquad ⑮.$$

⑭.⑮相对比

$$\frac{1}{\sqrt{1-2rx+r^2}}=\sum_{l=0}^{\infty}A_l r^l P_l(x) \qquad ⑯.$$

令 $x=1$. 由 $P_l(x)$ 性质. $P_l(1)\equiv 1$

$$\Rightarrow \frac{1}{1-r}=\sum_{l=0}^{\infty}A_l r^l \qquad ⑰.$$

⑰与 $f(r)=\frac{1}{1-r}=1+r+r^2+r^3+\cdots$.

$\therefore A_l=1,\quad l=0,1,2,\cdots$

提. $\boxed{\frac{1}{\sqrt{1-2rx+r^2}}=\sum_{l=0}^{\infty}r^l P_l(x)}$, $|r|<1$. ⑱

故 $\frac{1}{\sqrt{1-2rx+r^2}}$ 为 P_l 母函数 . #

另 $(l+1)P_{l+1}(x)-(2l+1)xP_l(x)+lP_{l-1}(x)=0$. #

(由 ⑱ 对 r 求导即得).

正交归一性:

$$\int_{-1}^{1}P_l(x)P_k(x)\,dx=\frac{2}{2l+1}\delta_{kl} \qquad \#.$$

• δ函数:

定义: 1. $\delta(x-x_0)=\begin{cases}0 & x\ne x_0\\ \infty & x=x_0.\end{cases}$

2. $\int_{-\infty}^{+\infty}\delta(x-x_0)\,dx=1$.

性质: 1. $\int_{-\infty}^{+\infty}f(x)\,\delta(x)\,dx=f(0)$,

2. 记 $\delta^{(n)}(x-a)=\frac{d^n}{dx^n}\delta(x-a)$.

有 $\int_{-\infty}^{+\infty}f(x)\,\delta^{(n)}(x-a)\,dx=(-)^n f^{(n)}(a)$.

(3) $\delta[\varphi(x)]=\sum_{i=1}^{k}\frac{\delta(x-x_i)}{|\varphi'(x_i)|}$

其中. x_i 为 $\varphi(x)=0$ 的单根 .

拓: 高维空间δ函数:

$$\begin{cases}\delta(M-M_0)=\begin{cases}0 & M\ne M_0\\ \infty & M=M_0.\end{cases}\\ \iiint_{-\infty}^{+\infty}\delta(M-M_0)\,dv=1\end{cases}$$

其中 $\delta(M-M_0)=\delta(x-x_0,y-y_0,z-z_0)$

$=\delta(x-x_0)\cdot\delta(y-y_0)\cdot\delta(z-z_0)$.

有 $\iiint f(M)\,\delta(M-M_0)\,dV=f(M_0)$.

※ $\int_{-\infty}^{+\infty}dt\; e^{i(\omega-\omega')t}=2\pi\,\delta(\omega-\omega')$.

δ函数常用积分表达式:

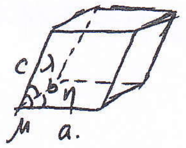
$$\delta(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{ikx}\,dk .$$

$$\delta(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\cos kx\,dk .$$

晶体的X射线衍射实验.

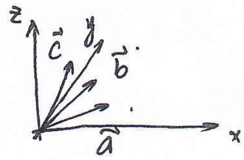
在学习化学选必二时，我们遇到了晶体结构测定的相关内容，下面就对X射线衍射实验进行探究。

在化学中，对平行六面体状晶胞的描述：



①

为方便计算，下面用空间坐标系重新描述：



$$\vec{a} = (a_1, a_2, 0)$$
$$\vec{b} = (b_1, b_2, 0) \quad ②$$
$$\vec{c} = (c_1, c_2, c_3).$$

①②间对应关系为：

$$a = \sqrt{a_1^2 + a_2^2}$$
$$b = \sqrt{b_1^2 + b_2^2}$$
$$c = \sqrt{c_1^2 + c_2^2 + c_3^2}$$
$$\cos\lambda = \frac{\vec{b}\cdot\vec{c}}{|\vec{b}|\cdot|\vec{c}|} = \frac{b_1c_1 + b_2c_2}{\sqrt{b_1^2+b_2^2}\cdot\sqrt{c_1^2+c_2^2+c_3^2}} \quad ③$$
$$\cos\mu = \frac{\vec{a}\cdot\vec{c}}{|\vec{a}||\vec{c}|} = \frac{a_1c_1 + a_2c_2}{\sqrt{a_1^2+a_2^2}\cdot\sqrt{c_1^2+c_2^2+c_3^2}}$$
$$\cos\eta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{a_1b_1 + a_2b_2}{\sqrt{a_1^2+a_2^2}\cdot\sqrt{b_1^2+b_2^2}}$$

* ② 表述较 ① 表述表面上多出一个自由度，实际运算中可将 a_2 取为0.

接下来计算光强分布：

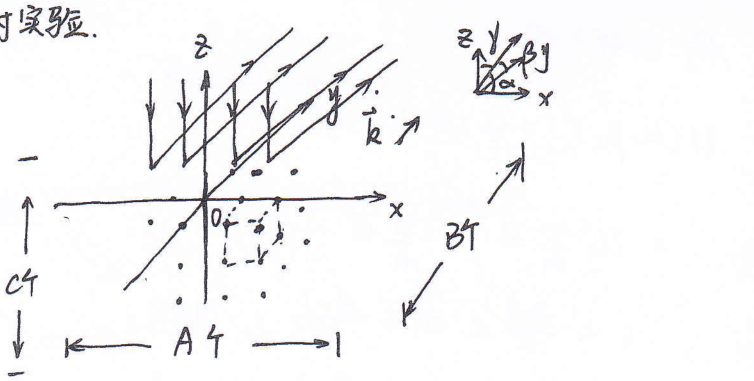
为便于计算，对模型做出如下简化处理：

(i) 认为实验用晶体沿某一晶胞平面打磨光滑.

(ii) 认为晶体晶胞数目为有限个.

(iii) 近似认为原子线度远小于晶格常数 a.b.c. 以致其与光的相互作用可用点近似处理.

*(iii) 应该为误差主要来源



记光的波矢 $\vec{k}$ 的方向为 $\hat{k} = (\cos\alpha, \cos\beta, \cos\gamma)$ #

$\{\vec{a}, \vec{b}, \vec{c}\}$ 方向原子数目为 $\{A, B, C\}$

则编号为 (m, n, l) 的原子位矢为：

$$\vec{r}_i = m\vec{a} + n\vec{b} + l\vec{c} \quad ④$$

以原点处原子为基准，光程差：

$$\Delta L_i = -\vec{r}_i \cdot (\hat{z} + \hat{k}) \quad ⑤$$

又有：$\Delta\varphi_i = k\Delta L_i$ ⑥

故振幅：

$$u(\alpha, \beta, \gamma) = \sum_i u_0 e^{i\Delta\varphi_i} \quad ⑦$$

将 ②④# 式代入 ⑤ ⇒.

$$(-\Delta L_i) = m(a_1\cos\alpha + a_2\cos\beta) + n(b_1\cos\alpha + b_2\cos\beta) + l(c_1\cos\alpha + c_2\cos\beta + c_3(\cos\gamma + 1)) \quad ⑧$$

⑥⑧ ⇒

$$-\Delta\varphi_i = k(a_1\cos\alpha + a_2\cos\beta)m + k(b_1\cos\alpha + b_2\cos\beta)n + k(c_1\cos\alpha + c_2\cos\beta + c_3(\cos\gamma + 1))l \quad ⑨$$

记：
$$\begin{cases} M = k(a_1\cos\alpha + a_2\cos\beta) \\ N = k(b_1\cos\alpha + b_2\cos\beta) \\ L = k(c_1\cos\alpha + c_2\cos\beta + c_3(\cos\gamma + 1)) \end{cases} \quad ⑩$$

则 ⑨ 可简写为：

$$-\Delta\varphi_i = Mm + Nn + Ll \quad ⑪$$

将 ⑪ 代入 ⑦ ⇒

$$U(\alpha,\beta,\gamma) = \sum_i U_0 e^{-i(Mm+Nn+Ll)}$$

$$= U_0 \sum_m e^{-iMm} \sum_n e^{-iNn} \sum_l e^{-iLl} \quad ⑫$$

$$(-\frac{A-1}{2} \le m \le \frac{A-1}{2}, \quad m\in Z$$

$$-\frac{B-1}{2} \le n \le \frac{B-1}{2}, \quad n\in Z$$

$$0 \le l \le C-1, \quad 且 l\in Z).$$

经计算：

$$\sum_m e^{-iMm} = \frac{\sin\frac{MA}{2}}{\sin\frac{M}{2}}$$

$$\sum_n e^{-iNn} = \frac{\sin\frac{NB}{2}}{\sin\frac{N}{2}} \quad ⑬$$

$$\sum_l e^{-iLl} = \frac{\sin\frac{LC}{2}}{\sin\frac{L}{2}} \cdot e^{-i\frac{L(C-1)}{2}}$$

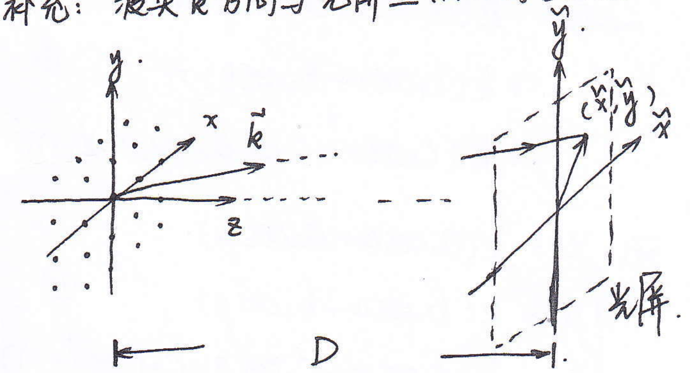
又∵ $I(\alpha,\beta,\gamma) = \frac{1}{2} U(\alpha,\beta,\gamma)\cdot U^*(\alpha,\beta,\gamma)$ ⑭

将 ⑫ ⑬ 代入 ⑭ ⇒

$$I(\alpha,\beta,\gamma) = I_0 \left(\frac{\sin\frac{MA}{2}}{\sin\frac{M}{2}}\right)^2 \left(\frac{\sin\frac{NB}{2}}{\sin\frac{N}{2}}\right)^2 \left(\frac{\sin\frac{LC}{2}}{\sin\frac{L}{2}}\right)^2$$

-------- ⑮

补充：波矢 $\vec{k}$ 方向与光屏上点的对应关系：



$$\begin{cases} \check{x} = D\tan\alpha \\ \check{y} = D\tan\beta \end{cases} \quad ⑯.$$

$(\check{x},\check{y})$ 为光屏位置坐标，D为晶体与光屏距离.

将计算结果与教材 P75 图 3-14 比较发现，晶态 SiO_2 的衍射图谱与计算结果定性符合. 表明这种计算方法也可以定性解释非简单立方的衍射图谱.

同时，计算过程中晶胞整齐排列带来的规则衍射图样一定程度上也是晶体各向异性的表现.

注：计算 ⑧ 时用到的式子：

$$\vec{a}\cdot\hat{z} = 0, \quad \vec{b}\cdot\hat{z} = 0, \quad \vec{c}\cdot\hat{z} = C_3$$

$$\vec{a}\cdot\hat{k} = a_1\cos\alpha + a_2\cos\beta$$

$$\vec{b}\cdot\hat{k} = b_1\cos\alpha + b_2\cos\beta$$

$$\vec{c}\cdot\hat{k} = c_1\cos\alpha + c_2\cos\beta + c_3\cos\gamma$$

其中，$\hat{z} = (0,0,1)$

$\hat{k} = (\cos\alpha, \cos\beta, \cos\gamma)$.

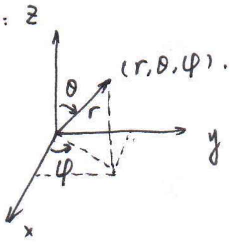
$\alpha.\beta.\gamma$ 满足：

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

雷逸鸣

于 2023年春.

法一:



几何关系:

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases} \quad ①$$

由①:

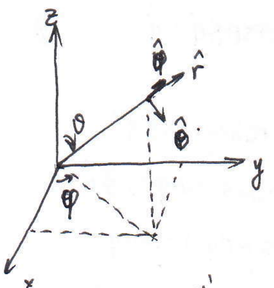
$$\begin{cases} \dot{x} = \dot{r}\sin\theta\cos\varphi + r\cos\theta\cos\varphi\,\dot\theta - r\sin\theta\sin\varphi\,\dot\varphi \\ \dot{y} = \dot{r}\sin\theta\sin\varphi + r\cos\theta\sin\varphi\,\dot\theta + r\sin\theta\cos\varphi\,\dot\varphi \\ \dot{z} = \dot{r}\cos\theta - r\sin\theta\,\dot\theta \end{cases} \quad ②$$

由②:

$$\begin{aligned} \ddot{x} = {} & \ddot{r}\sin\theta\cos\varphi + \dot{r}\cos\theta\cos\varphi\,\dot\theta - \dot{r}\sin\theta\sin\varphi\,\dot\varphi \\ & + \dot{r}\cos\theta\cos\varphi\,\dot\theta + r\cos\theta\cos\varphi\,\ddot\theta - r\sin\theta\cos\varphi\,\dot\theta^2 \\ & - r\cos\theta\sin\varphi\,\dot\theta\dot\varphi - \dot{r}\sin\theta\sin\varphi\,\dot\varphi - r\sin\theta\sin\varphi\,\ddot\varphi \\ & - r\cos\theta\sin\varphi\,\dot\varphi\dot\theta - r\sin\theta\cos\varphi\,\dot\varphi^2 \quad ③ \\ = {} & -(2\dot{r}\dot\varphi + r\ddot\varphi)\sin\theta\sin\varphi + (\ddot{r} - r\dot\theta^2 - r\dot\varphi^2)\sin\theta\cos\varphi \\ & - 2\dot\theta\dot\varphi\, r\cos\theta\sin\varphi + (2\dot{r}\dot\theta + r\ddot\theta)\cos\theta\cos\varphi. \end{aligned}$$

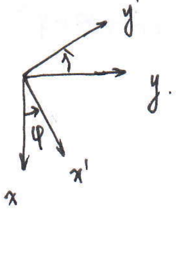
$$\begin{aligned} \ddot{y} = {} & \ddot{r}\sin\theta\sin\varphi + \dot{r}\sin\theta\cos\varphi\,\dot\varphi + \dot{r}\cos\theta\sin\varphi\,\dot\theta \\ & + \dot{r}\cos\theta\sin\varphi\,\dot\theta + r\cos\theta\cos\varphi\,\dot\theta\dot\varphi + r\cos\theta\sin\varphi\,\ddot\theta \\ & - r\sin\theta\sin\varphi\,\dot\theta^2 + \dot{r}\sin\theta\cos\varphi\,\dot\varphi + r\cos\theta\cos\varphi\,\dot\theta\dot\varphi \\ & - r\sin\theta\sin\varphi\,\dot\varphi^2 + r\sin\theta\cos\varphi\,\ddot\varphi \\ = {} & (\ddot{r} - r\dot\theta^2 - r\dot\varphi^2)\sin\theta\sin\varphi. \\ & + (2\dot{r}\dot\varphi + r\ddot\varphi)\sin\theta\cos\varphi. \\ & + (2\dot{r}\dot\theta + r\ddot\theta)\cos\theta\sin\varphi. \\ & + 2\dot\theta\dot\varphi\, r\cos\theta\cos\varphi. \quad ④ \end{aligned}$$

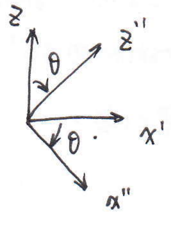
$$\begin{aligned} \ddot{z} = {} & \ddot{r}\cos\theta - \dot{r}\sin\theta\,\dot\theta - \dot{r}\sin\theta\,\dot\theta \\ & - r\cos\theta\,\dot\theta^2 - r\sin\theta\,\ddot\theta \quad ⑤ \\ = {} & (\ddot{r} - r\dot\theta^2)\cos\theta - (r\ddot\theta + 2\dot{r}\dot\theta)\sin\theta. \end{aligned}$$



$$\begin{aligned} \vec{a} &= a_x\hat{x} + a_y\hat{y} + a_z\hat{z} \\ &= a_r\hat{r} + a_\theta\hat\theta + a_\varphi\hat\varphi. \end{aligned} \quad ⑥$$

$$\begin{pmatrix} \hat\theta \\ \hat\varphi \\ \hat{r} \end{pmatrix} = \vec{R}(\theta)\,\vec{R}(\varphi) \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} \quad ⑦.$$





$$\vec{R}(\varphi) = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ⑧$$

$$\vec{R}(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \quad ⑨$$

由⑦⇒ $\vec{a} = (a_x, a_y, a_z)\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}$

$$= (a_x, a_y, a_z)\ \vec{R}^{-1}(\varphi)\,\vec{R}^{-1}(\theta)\begin{bmatrix} \hat\theta \\ \hat\varphi \\ \hat{r} \end{bmatrix} \quad ⑩.$$

$$\vec{a} = (a_\theta, a_\varphi, a_r)\begin{bmatrix} \hat\theta \\ \hat\varphi \\ \hat{r} \end{bmatrix}. \quad ⑪$$

由⑩⑪⇒

$$(a_\theta, a_\varphi, a_r) = (a_x, a_y, a_z)\ \vec{R}^{-1}(\varphi)\,\vec{R}^{-1}(\theta). \quad ⑫$$

$$\vec{R}^{-1}(\varphi) = \vec{R}(-\varphi). \quad \vec{R}^{-1}(\theta) = \vec{R}(-\theta) \quad ⑬$$

记 $\vec{T}^{-1}(\theta,\varphi) = \vec{R}^{-1}(\varphi)\,\vec{R}^{-1}(\theta)$, 则:

$$\begin{aligned} \vec{T}^{-1}(\theta,\varphi) &= \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta\cos\varphi & -\sin\varphi & \sin\theta\cos\varphi \\ \cos\theta\sin\varphi & \cos\varphi & \sin\theta\sin\varphi. \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad ⑭. \end{aligned}$$

⑭代入⑫.

$$\begin{cases} a_\theta = \cos\theta\cos\varphi\, a_x + \cos\theta\sin\varphi\, a_y - \sin\theta\, a_z. \\ a_\varphi = -\sin\varphi\, a_x + \cos\varphi\, a_y \\ a_r = \sin\theta\cos\varphi\, a_x + \sin\theta\sin\varphi\, a_y + \cos\theta\, a_z. \end{cases} \quad ⑮$$

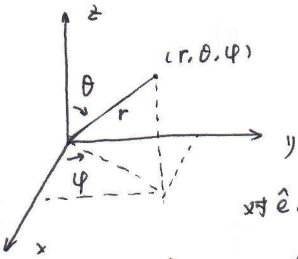
将③④⑤代入⑮得:

$$\begin{cases} a_\theta = (r\ddot\theta + 2\dot{r}\dot\theta)\cancel{\sin^2\theta} - r\dot\varphi^2\sin\theta\cos\theta \\ a_\varphi = (r\ddot\varphi + 2\dot{r}\dot\varphi)\sin\theta + 2r\dot\theta\dot\varphi\cos\theta. \\ a_r = \ddot{r} - r\dot\theta^2 - r\dot\varphi^2\sin^2\theta \end{cases} \quad ⑯.$$

BY建议:

$$\begin{aligned} a_\varphi &= r\sin\theta\,\ddot\varphi + 2\dot{r}\sin\theta\,\dot\varphi + 2r\dot\theta\cos\theta\,\dot\varphi \\ &= r\sin\theta\,\ddot\varphi + 2\,(\dot{r\sin\theta})\,\dot\varphi \end{aligned}$$

法二：



$$\begin{cases}\hat{r}=\hat{r}(\theta,\varphi)\\ \hat{\theta}=\hat{\theta}(\theta,\varphi)\\ \hat{\varphi}=\hat{\varphi}(\theta,\varphi)\end{cases} \quad ①.$$

对 $\hat{e}$，有 $d\hat{e}=\frac{\partial \hat{e}}{\partial\theta}d\theta+\frac{\partial\hat{e}}{\partial\varphi}d\varphi$ ②.

$$\cancel{又:\ \frac{\partial\hat r}{\partial\theta}=\hat\theta \quad \frac{\partial\hat r}{\partial\varphi}=\hat\varphi \quad \frac{\partial\hat\theta}{\partial\theta}=-\hat r} \quad ③.$$
$$\cancel{\frac{\partial\hat\theta}{\partial\varphi}=\hat\varphi \quad \frac{\partial\hat\varphi}{\partial\theta}=\hat\theta \quad \frac{\partial\hat\varphi}{\partial\varphi}=-\hat r}$$

①∪③ ⇒ .

$$\cancel{\dot{\hat r}=\frac{d\hat r}{dt}=\frac{\partial\hat r}{\partial\theta}\frac{d\theta}{dt}+\frac{\partial\hat r}{\partial\varphi}\frac{d\varphi}{dt}=\dot\theta\hat\theta+\dot\varphi\hat\varphi}$$

由
$$\begin{bmatrix}\hat\theta\\ \hat\varphi\\ \hat r\end{bmatrix}=\vec{R}(\theta)\,\vec{R}(\varphi)\begin{bmatrix}\hat x\\ \hat y\\ \hat z\end{bmatrix}=\vec{\vec{T}}(\theta,\varphi)\begin{bmatrix}\hat x\\ \hat y\\ \hat z\end{bmatrix} \quad ④$$

$$\frac{d}{dt}\begin{bmatrix}\hat\theta\\ \hat\varphi\\ \hat r\end{bmatrix}=\left[\frac{d}{dt}\vec{\vec{T}}(\theta,\varphi)\right]\begin{bmatrix}\hat x\\ \hat y\\ \hat z\end{bmatrix} \quad ⑤.$$

$$\left(由\ \frac{d}{dt}\begin{bmatrix}\hat x\\ \hat y\\ \hat z\end{bmatrix}=\vec{0}\right).$$

$$\therefore\ \frac{d}{dt}\begin{bmatrix}\hat\theta\\ \hat\varphi\\ \hat r\end{bmatrix}=\left[\frac{d}{dt}\vec{\vec{T}}(\theta,\varphi)\right]\vec{\vec{T}}^{-1}(\theta,\varphi)\begin{bmatrix}\hat\theta\\ \hat\varphi\\ \hat r\end{bmatrix} \quad ⑥$$

经计算得：

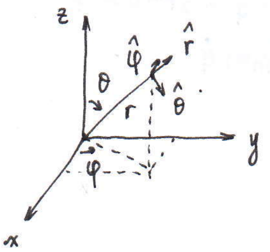
$$\begin{bmatrix}\dot{\hat\theta}\\ \dot{\hat\varphi}\\ \dot{\hat r}\end{bmatrix}=\begin{bmatrix}0 & \cos\theta\dot\varphi & -\dot\theta\\ -\cos\theta\dot\varphi & 0 & -\sin\theta\dot\varphi\\ \dot\theta & \sin\theta\dot\varphi & 0\end{bmatrix}\begin{bmatrix}\hat\theta\\ \hat\varphi\\ \hat r\end{bmatrix} \quad ⑦.$$

即
$$\begin{cases}\dot{\hat\theta}=\cos\theta\,\dot\varphi\,\hat\varphi-\dot\theta\,\hat r\\ \dot{\hat\varphi}=-\cos\theta\,\dot\varphi\,\hat\theta-\sin\theta\,\dot\varphi\,\hat r\\ \dot{\hat r}=\dot\theta\,\hat\theta+\sin\theta\,\dot\varphi\,\hat\varphi\end{cases} \quad ⑧.$$

亦即上述判去的③应改为：

$$\frac{\partial\hat r}{\partial\theta}=\hat\theta \quad \frac{\partial\hat r}{\partial\varphi}=\sin\theta\,\hat\varphi \quad \frac{\partial\hat\theta}{\partial\varphi}=\cos\theta\,\hat\varphi$$
$$\frac{\partial\hat\theta}{\partial\theta}=-\hat r \quad \frac{\partial\hat\varphi}{\partial\varphi}=-\cos\theta\,\hat\theta-\sin\theta\,\hat r \quad \frac{\partial\hat\varphi}{\partial\theta}=0 \quad ⑨.$$

*：⑨式实际上可由几何在图中直接推出，此处采用几何法作为检验.☺



于是由⑧式⇒

$$\vec r=r\hat r \quad ⑩.$$

$$\vec v=\dot{\vec r}=\dot r\hat r+r\dot{\hat r} \quad ⑪.$$
$$=\dot r\hat r+r\dot\theta\hat\theta+r\sin\theta\dot\varphi\hat\varphi$$

即 $v_r=\dot r$，$v_\theta=r\dot\theta$，$v_\varphi=r\sin\theta\dot\varphi$ （正确）.

$$\vec a=\dot{\vec v}=\ddot r\hat r+\dot r\dot{\hat r}+\dot r\dot\theta\hat\theta+r\ddot\theta\hat\theta+r\dot\theta\dot{\hat\theta}+\dot r\sin\theta\dot\varphi\hat\varphi+r\cos\theta\dot\theta\dot\varphi\hat\varphi+r\sin\theta\ddot\varphi\hat\varphi+r\sin\theta\dot\varphi\dot{\hat\varphi}$$
$$=\ddot r\hat r+\dot r\dot\theta\hat\theta+\dot r\sin\theta\dot\varphi\hat\varphi+\dot r\dot\theta\hat\theta+r\ddot\theta\hat\theta+r\cos\theta\dot\theta\dot\varphi\hat\varphi-r\dot\theta^2\hat r+\dot r\sin\theta\dot\varphi\hat\varphi+r\cos\theta\dot\theta\dot\varphi\hat\varphi+r\sin\theta\ddot\varphi\hat\varphi+r\sin\theta\dot\varphi(-\cos\theta\dot\varphi\hat\theta-\sin\theta\dot\varphi\hat r)$$
$$=(\ddot r-r\dot\theta^2-r\sin^2\theta\dot\varphi^2)\hat r+(r\ddot\theta+2\dot r\dot\theta-r\sin\theta\cos\theta\dot\varphi^2)\hat\theta+(r\sin\theta\ddot\varphi+2\dot r\dot\varphi\sin\theta+2r\cos\theta\dot\theta\dot\varphi)\hat\varphi \quad ⑫.$$

$$\Rightarrow\begin{cases}a_r=\ddot r-r\dot\theta^2-r\sin^2\theta\dot\varphi^2\\ a_\theta=r\ddot\theta+2\dot r\dot\theta-r\sin\theta\cos\theta\dot\varphi^2\\ a_\varphi=r\sin\theta\ddot\varphi+2\dot r\dot\varphi\sin\theta+2r\cos\theta\dot\theta\dot\varphi\end{cases} \quad ⑬.$$

与法一⑯式得到相同结果

下面用此方法推导一下"jerk"

$$\vec J=\dot{\vec a}=(\dddot r-\dot r\dot\theta^2-2r\dot\theta\ddot\theta-\dot r\sin^2\theta\dot\varphi^2-2r\sin\theta\cos\theta\dot\theta\dot\varphi^2-2r\sin^2\theta\dot\varphi\ddot\varphi)\hat r+(\ddot r-r\dot\theta^2-r\sin^2\theta\dot\varphi^2)\dot{\hat r}$$
$$+(\dot r\ddot\theta+r\dddot\theta+2\ddot r\dot\theta+2\dot r\ddot\theta-\dot r\sin\theta\cos\theta\dot\varphi^2-r\cos^2\theta\dot\theta\dot\varphi^2+r\sin^2\theta\dot\theta\dot\varphi^2-2r\sin\theta\cos\theta\dot\varphi\ddot\varphi)\hat\theta$$
$$+(r\ddot\theta+2\dot r\dot\theta-r\sin\theta\cos\theta\dot\varphi^2)\dot{\hat\theta}+(\dot r\sin\theta\ddot\varphi+r\cos\theta\dot\theta\ddot\varphi+r\sin\theta\dddot\varphi+2\ddot r\dot\varphi\sin\theta+2\dot r\ddot\varphi\sin\theta$$
$$+2\dot r\dot\varphi\cos\theta\dot\theta+2\dot r\cos\theta\dot\theta\dot\varphi-2r\sin\theta\dot\theta^2\dot\varphi+2r\cos\theta\ddot\theta\dot\varphi+2r\cos\theta\dot\theta\ddot\varphi)\hat\varphi$$
$$+(r\sin\theta\ddot\varphi+2\dot r\dot\varphi\sin\theta+2r\cos\theta\dot\theta\dot\varphi)\dot{\hat\varphi} \quad ⑭.$$

下面整理：

$$J_r=\dddot r-\dot r\dot\theta^2-2r\dot\theta\ddot\theta-\dot r\sin^2\theta\dot\varphi^2-2r\sin\theta\cos\theta\dot\theta\dot\varphi^2-2r\sin^2\theta\dot\varphi\ddot\varphi-(r\ddot\theta+2\dot r\dot\theta-r\sin\theta\cos\theta\dot\varphi^2)\dot\theta-(r\sin\theta\ddot\varphi+2\dot r\dot\varphi\sin\theta+2r\cos\theta\dot\theta\dot\varphi)\sin\theta\dot\varphi$$
$$=\dddot r-3\dot r\dot\theta^2-3r\dot\theta\ddot\theta-3r\sin^2\theta\dot\varphi\ddot\varphi-3\dot r\sin^2\theta\dot r\dot\varphi^2+r\sin\theta\cos\theta\dot\theta\dot\varphi^2$$

J_θ，J_φ 同理.<略>

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1. 运动学

参考系

坐标系　直角坐标系

※平面极坐标 ※自然坐标系

矢量和标量

质点运动的位移和路程 速度 加速度

匀速及匀变速直线运动及其图像

运动的合成与分解 抛体运动 圆周运动

圆周运动中的切向加速度和法向加速度

曲率半径 角速度和※角加速度

相对运动 伽里略速度变换

2.动力学

重力 弹性力 摩擦力

惯性参考系

牛顿第一、二、三运动定律 胡克定律万有引力定律

均匀球壳对壳内和壳外质点的引力公式（不要求导出）

※非惯性参考系 ※平动加速参考系中的惯性力

※匀速转动参考系惯性离心力、视重

☆科里奥利力 $-2m\vec{\omega}\times\vec{v}$

3.物体的平衡

共点力作用下物体的平衡

力矩 刚体的平衡条件

☆虚功原理

4.动量

冲量 动量 质点与质点组的动量定理

动量守恒定律

※质心 ※质心运动定理

※质心参考系

反冲运动

※变质量体系的运动

5.机械能

功和功率

动能和动能定理 ※质心动能定理

重力势能 引力势能

质点及均匀球壳壳内和壳外的引力势能公式

（不要求导出）

弹簧的弹性势能

功能原理 机械能守恒定律

碰撞

弹性碰撞与非弹性碰撞 恢复系数

6.※角动量

冲量矩 角动量

质点和质点组的角动量定理和转动定理

角动量守恒定律

7.有心运动

在万有引力和库仑力作用下物体的运动

开普勒定律

行星和人造天体的圆轨道和椭圆轨道运动

8.※刚体

刚体的平动 刚体的定轴转动

刚体绕轴的转动惯量

平行轴定理正交轴定理

质心轴为基准

刚体定轴转动的角动量定理 刚体的平面平行运动

9.流体力学

静止流体中的压强

浮力

$\oiint \rho \vec{v} \cdot d\vec{s} + \iiint \frac{\partial \rho}{\partial t} dV = 0$.

☆连续性方程 ☆伯努利方程

$p + \frac{1}{2}\rho v^2 + \rho g h = const$.

10.振动

简谐振动 振幅 频率和周期 相位

振动的图像

参考圆 简谐振动的速度

(线性)恢复力 由动力学方程确定简谐振动的频率

简谐振动的能量

同方向同频率简谐振动的合成

阻尼振动 受迫振动和共振(定性了解)

11.波动

横波和纵波

波长 频率和波速的关系

波的图像

※平面简谐波的表示式

波的干涉 ※驻波 波的衍射(定性)

声波 声音的响度、音调和音品

声音的共鸣 乐音和噪声(前3项均不要求定量计算)

※多普勒效应

热学

1. 分子动理论 $\square$

原子和分子大小的数量级 $1\mathring{A} = 10^{-10}m$.

分子的热运动和碰撞 布朗运动

※压强的统计解释 $P = nkT$: Maxwell

☆麦克斯韦速率分布的定量计算;

※分子热运动自由度 ※能均分定理;

$\square$温度的微观意义 $\bar{\varepsilon} = \frac{3}{2}kT$.

分子热运动的动能

※气体分子的平均平动动能

分子力 分子间的势能

物体的内能

2.气体的性质

温标 热力学温标

气体实验定律 理想气体状态方程

道尔顿分压定律

混合理想气体状态方程

理想气体状态方程的微观解释(定性)

3.热力学第一定律

热力学第一定律

理想气体的内能

热力学第一定律在理想气体等容、等压、等温、

绝热过程中的应用

※多方过程及应用

※定容热容量和定压热容量

※绝热过程方程

※等温、绝热过程中的功

※热机及其效率 ※卡诺定理

4.热力学第二定律

※热力学第二定律的开尔文表述和克劳修斯表述

※可逆过程与不可逆过程

※宏观热力学过程的不可逆性

※理想气体的自由膨胀

※热力学第二定律的统计意义 $S = \nu R \ln V + \nu C_V \ln T$.

☆热力学第二定律的数学表达式

☆熵、熵增

5.液体的性质

液体分子运动的特点

表面张力系数

※球形液面两边的压强差

浸润现象和毛细现象(定性)

6.固体的性质

晶体和非晶体 空间点阵 ×

固体分子运动的特点

7.物态变化

热一: $dU = -pdV + dQ$.

准静态 ⇒ $dS = \frac{dQ}{T}$. $dQ = TdS$.

⇒ $dU = -pdV + TdS$.

$\therefore \left(\frac{\partial U}{\partial V}\right)_S = -p$; $\left(\frac{\partial U}{\partial S}\right)_V = T$; $-\left(\frac{\partial p}{\partial S}\right)_V = \left(\frac{\partial T}{\partial V}\right)_S$.

$pdV + Vdp = d(pV)$.

$dU = -d(pV) + Vdp + TdS$.

$dH = d(U+pV) = Vdp + TdS$.

$f(x,y)$

$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$.

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

$d(TS) = TdS + SdT$.

$A = U - TS$.

$G = U + PV - TS$.

$\frac{dp}{dT} = \frac{L}{T(V_g - V_l)}$

熔化和凝固　熔点　熔化热

蒸发和凝结　饱和气压　沸腾和沸点

汽化热　临界温度

固体的升华

空气的湿度和湿度计　露点

8.热传递的方式

传导　※导热系数

对流

辐射　※黑体辐射的概念　※斯忒藩定律　σT^4.

※维恩位移定律　$\lambda_m = \frac{b}{T}$

9.热膨胀

热膨胀和膨胀系数　$\propto \rho$.

电磁学

1.静电场

电荷守恒定律

库仑定律

电场强度　电场线

点电荷的场强　场强叠加原理

匀强电场

均匀带电球壳内、外的场强公式(不要求导出)

※高斯定理及其在对称带电体系中的应用

电势和电势差　等势面

点电荷电场的电势

电势叠加原理

均匀带电球壳内、外的电势公式

电场中的导体　静电屏蔽

※静电镜像法

电容　平行板电容器的电容公式

※球形、圆柱形电容器的电容

电容器的连联接

※电荷体系的静电能，※电场的能量密度，

电容器充电后的电能

☆电偶极矩

☆电偶极子的电场和电势

电介质的概念

☆电介质的极化与极化电荷

☆电位移矢量

2.稳恒电流

欧姆定律　电阻率和温度的关系

电功和电功率

电阻的串、并联

电动势　闭合电路的欧姆定律

一段含源电路的欧姆定律　※基尔霍夫定律　数学公式

电流表　电压表　欧姆表

惠斯通电桥

补偿电路

3.物质的导电性

金属中的电流　欧姆定律的微观解释

※液体中的电流　※法拉第电解定律

※气体中的电流　※被激放电和自激放电(定性)

真空中的电流　示波器

半导体的导电特性　p 型半导体和 n 型半导体　※P-N 结

晶体二极管的单向导电性※及其微观解释(定性)

三极管的放大作用(不要求掌握机理)

超导现象　☆超导体的基本性质

4.磁场

电流的磁场　※毕奥-萨伐尔定律

磁场叠加原理

磁感应强度　磁感线

匀强磁场

长直导线、圆线圈、螺线管中的电流的磁场分布(定性)

※安培环路定理及在对称电流体系中的应用

※圆线圈中的电流在轴线上和环面上的磁场

$\frac{\mu_0 I R^2}{2(R^2+r^2)^{3/2}}$

☆磁矩

安培力 洛伦兹力 带电粒子荷质比的测定

质谱仪 回旋加速器 霍尔效应

5. 电磁感应

法拉第电磁感应定律

楞次定律

※感应电场(涡旋电场)

自感和互感 自感系数

※通电线圈的自感磁能(不要求推导)

6.交流电

交流发电机原理 交流电的最大值和有效值

$\frac{1}{\sqrt{2}}$

☆交流电的矢量和复数表述

纯电阻、纯电感、纯电容电路 感抗和容抗

※电流和电压的相位差

整流 滤波和稳压

☆谐振电路☆交流电的功率

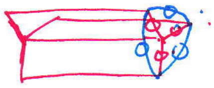
$\bar{P}=IU\cos\varphi$. $I=\frac{\sqrt{2}}{2}I_m$ $U=\frac{\sqrt{2}}{2}U_m$.

$P=\frac{1}{2}Re(\tilde{U}\tilde{I}^*)$

$=\frac{1}{2}\tilde{U}\tilde{I}^*=\frac{1}{2}\tilde{U}^*\tilde{I}$.

☆三相交流电及其连接法

☆感应电动机原理



Y. Δ.

理想变压器

远距离输电

7.电磁振荡和电磁波

电磁振荡 振荡电路及振荡频率 赫兹实验

$\varepsilon=\frac{1}{2}\varepsilon_0E^2+\frac{1}{2}\frac{B^2}{\mu_0}$

电磁场和电磁波

☆电磁场能量密度、能流密度

$\vec{S}=\vec{E}\times\vec{H}$ $\vec{g}=\vec{D}\times\vec{B}$ $\vec{l}=\vec{r}\times\vec{g}$.

电磁波的波速 电磁波谱

电磁波的发射和调制 电磁波的接收、调谐、检波

光学

1. 几何光学

※费马原理

光的传播 反射 折射 全反射

光的色散 折射率与光速的关系 $n=\frac{c}{v}$

平面镜成像 球面镜成像公式及作图法

※球面折射成像公式 ※焦距与折射率、球面半径的关系

薄透镜成像公式及作图法

眼睛 放大镜 显微镜 望远镜

※其它常用光学仪器

2.波动光学

光程

※惠更斯原理(定性)

光的干涉现象 双缝干涉

光的衍射现象

※夫琅禾费衍射

※光栅 ※布拉格公式

※分辨本领(不要求导出)

光谱和光谱分析(定性)

※光的偏振※自然光与偏振光

※马吕斯定律※布儒斯特定律

近代物理

1.光的本性

光电效应 ※康普顿散射

光的波粒二象性　光子的能量与动量

2.原子结构

卢瑟福实验　原子的核式结构

玻尔模型

用玻尔模型解释氢光谱

※用玻尔模型解释类氢光谱

原子的受激辐射　激光的产生(定性)和特性

3.原子核

原子核的尺度数量级

天然放射性现象　原子核的衰变　半衰期

放射线的探测

质子的发现　中子的发现　原子核的组成

核反应方程

质能关系式　裂变和聚变　质量亏损

4.粒子

"基本粒子"轻子与夸克(简单知识)

四种基本相互作用

弱相互作用?

实物粒子具有波粒二象性

※物质波

※德布罗意关系

$\lambda = \frac{h}{p}$.

※不确定关系

$\Delta p \cdot \Delta x \geq \hbar/2$　$\Delta E \cdot \Delta t \geq \hbar/2$

5.※狭义相对论

爱因斯坦假设

洛伦兹变换

时间和长度的相对论效应多普勒效应

☆速度变换

相对论动量　相对论能量　相对论动能

相对论动量和能量关系

6.※太阳系，银河系，宇宙和黑洞的初步知识.

单位制

国际单位制与量纲分析

数学基础

1.中学阶段全部初等数学(包括解析几何).

2.矢量的合成和分解，矢量的运算，极限、无限大和无限小的初步概念.

3.※微积分初步及其应用：

含一元微积分的简单规则；

微分：包括多项式、三角函数、指数函数、对数函数的导数，函数乘积和商的导数，复合函数的导数。

积分：包括多项式、三角函数、指数函数、对数函数的简单积分。

$\int \log_x a \cdot dx$.

$= \ln a \cdot \int \frac{dx}{\ln x}$.

LG 考前易错整理

1. 努力把题做对.

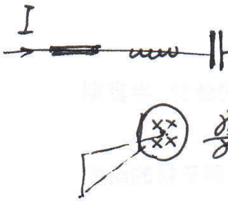
   做对一题 比骗两题分更有效!

2. 二次方程等算得多解. 舍解需标明原因.

3. 光学题特殊点 ($u=0$. $u=+\infty$. $u=f$ ·2f).

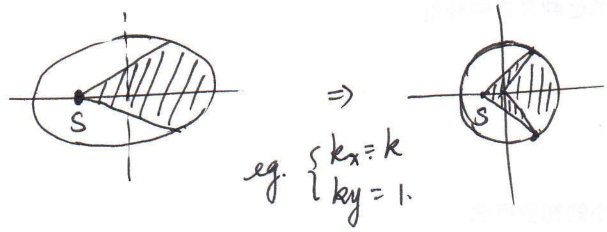
4. 电势差 $U_{AB}=\varphi_A-\varphi_B=\frac{1}{C}q+IR+L\dot{I}-\varepsilon$

   $U=-\varepsilon$. 感生电场对电势"贡献"



亮次.

5. 天体扫过椭圆面积 — 放缩.



* 演示. (天体不位于椭圆中心之外).

6. 干涉条纹 CASIO 打表只可估测. 精确需继续打表.

7. $\omega_{进}=\omega_r-\omega_\theta$.

8. 简谐运动 初相位 — 平衡位置关联. $\varphi=k\cdot\frac{\pi}{2}$ 时, 再慎重考虑一下. 小心有坑.

9. 符号与题干保持一致.

10. 滚钢笔. 卡棱. 滚上绝对粗糙面……

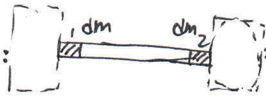
    使用角支点角动量守恒!!!

11. 题干给极值 / m.M 比例类. 要代 特值!!! (亦简化过程).

12. 光needs干涉用夹角. ~~(一般不是不行. 但巨烦 非小量近似)~~.

13. 加速度关联有时可由速度关联. 求导而得.

14. 相对论: $v=\frac{pc^2}{E}$ (非光子粒子. 光子 $v=c$). 自洽

15. 长导管. 多孔塞. 非平衡:  微元. $P_i=P_i$. $T_i=T_i$. ($i=1,2$).
    + "连续性方程 or 能量"

16. "正则角动量. 正则动量" 守恒. Good!

17. 电像法: 补电荷闭守恒 实质上在调电势零点.

    不补电荷. 求 φ 需: $\varphi=\sum\left(\frac{kq_i}{r_i}\right)+\varphi_0$ → 由边界条件给定